# Multivariable Model Predictive Control Design for Turbocharged Exhaust Gas Recirculation System in Marine Combustion Engines

*Sergey Samokhin, Kai Zenger*
*Department of Electrical Engineering and Automation*
*Aalto University*
*Otaniementie 17, 02150 Espoo, Finland*

KEYWORDS marine engine, MPC, $H_\infty$ control, turbocharged EGR

## 1 ABSTRACT

This work investigates a novel turbocharged EGR system installed on a laboratory marine engine. A problem of recirculation of high and stable portion of exhaust gas in marine engines is addressed. Although conventional EGR systems have been extensively presented in research papers, the setup shown in this paper has not been well described. Therefore, we develop a mean-value model of the system and design a multivariable model predictive controller (MPC) for it. We also evaluate the advantages of a multivariable MPC in its application to marine diesel engines over other multivariable algorithms, for example $H_\infty$ controller.

## 2 INTRODUCTION

The emission reduction problem has become important during the past decades due to a significant growth of the transportation sector. It is, therefore, crucial for manufacturers to produce engines that can meet strict emission regulation requirements. Internal combustion engines used in ships act as sources of pollution gases, among which the following are known to be harmful: hydrocarbons, carbon dioxide ($CO_2$) and monoxide ($CO$), nitrogen oxides ($NO_x$) and particulate matter ($PM$) /10/.

This work describes modeling and control of the most powerful tool for $NO_x$ emission reduction, which is called the external exhaust gas recirculation (EGR) and was introduced in 1970's. Since $NO_x$ is formed at high in-cylinder temperatures, the basic idea is to reroute part of the engine exhaust gases back to the intake manifold to lower the combustion temperature inside the cylinders. The methodology turned out to be quite efficient and the variety of configurations was suggested by R&D companies to improve the EGR performance /2/, /3/. In the conventional EGR system, exhaust is delivered by connecting intake and exhaust manifolds with a hose and separating them with a controllable valve. This is a simple, yet reliable system, widely used in the production engines. However, it has a drawback of being incapable to provide a high and stable EGR flow at different operating points of the engine. Modeling and control of combustion engines equipped with conventional EGR system has been presented by many researchers, for example /5/, /7/, /8/, /15/ and /16/.

This work is inspired by a different type of EGR, namely turbocharged high-pressure exhaust gas recirculation (HPEGR) system (Fig. 1, left), which is a novel approach for a near-zero $NO_x$ emission achievement. Literature survey shows that there is a lack of knowledge on such turbocharged HPEGR, although several patents are available /1/, /12/. Therefore, in this paper we develop a mean value air-path engine model /4/ and propose a multivariable control system for it. There are two primary control target here:

- track the EGR fraction reference value during transients
- reject disturbances in VGT shaft speed to diminish exhaust gas mass flow oscillations

In this work, we assume that all the states and EGR are measured and no estimation is needed. The designed model has been validated with the data obtained in open-loop engine tests.

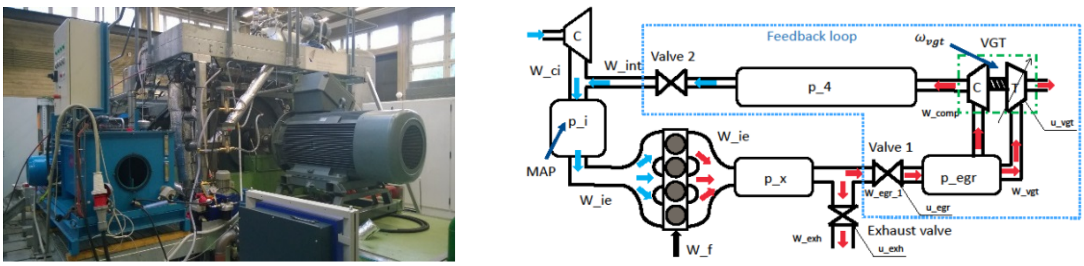


**Figure 1.** Extreme Value Engine enhanced with piping for high-pressure exhaust gas recirculation (left) and its modelling configuration (right).

In this article, we develop a model predictive controller (MPC), which, amongst the advanced control systems, has become the most popular algorithm used in the automotive industry. While being the extension to the classical optimal control, MPC contains a number of features making it attractive in the application for engine control. The most important aspect here is the constrained optimization that allows explicit handling of many important physically limited variables in the engine. A better engine performance can be achieved if these limitations are taken into account during the optimization process. Another, advantage of MPC is its ability to deal with the multi-input multi-output systems in a natural way and reduce the input/output interaction.

We also design the multivariable mixed-sensitivity $H_\infty$ and proportional integral (PI) controllers to evaluate the performance of the MPC in our application. The developed controllers are compared via simulations in Matlab/Simulink™.

We start the work by reviewing the engine mean-value model in Section 2. The controllers design is described in Section 3 and their numerical simulation is done in Section 4. The conclusion is done in Section 5.

## 2 ENGINE MODELING

We give a brief review of the mean-value model of the CI engine equipped with an HPEGR system developed in /11/. Modeling of similar systems has been presented by several authors, see for example /6/, /14/, /15/. The engine block diagram is shown in Fig. 1, right.

The EGR system mounted on the test engine has long connecting pipes and volumetric balancing vessels. We can therefore assume temperatures in control volumes constant for simplification.

The main dynamics of the system are defined by the pressure $p$ in the four control volumes, turbocharger shaft speed $\omega$ and the compressor power $P$:

$$\dot{p}_i(t) = \frac{R_i T_i}{V_i}\left(W_{ci}(t) + W_{egr,2}(t) - W_{ie}(t)\right) \tag{1a}$$

$$\dot{p}_x(t) = \frac{R_x T_x}{V_x}\left(W_{ie}(t) + W_f(t) - W_{exh}(t) - W_{egr,1}(t)\right) \tag{1b}$$

$$\dot{p}_{egr}(t) = \frac{R_x T_3}{V_{egr}}\left(W_{egr,1}(t) - W_{comp}(t) - W_{vgt}(t)\right) \tag{1c}$$

$$\dot{p}_4(t) = \frac{R_x T_5}{V_4}\left(W_{comp}(t) - W_{egr,2}(t)\right) \tag{1d}$$

$$\dot{\omega}_{tc}(t) = \frac{P_t(t) - P_c(t)}{J\omega_{tc}} \tag{1e}$$

$$\dot{P}_c(t) = \frac{1}{\tau_c}\left(P_t(t) - P_c(t)\right), \tag{1f}$$

where $R$, $T$ and $V$ denote the specific gas constant, its temperature and the manifold volume, respectively. The mass flows and powers are denoted as $W$ and $P$, respectively. The subscripts stand for the following engine components: *i*, *x*, *egr* and *4* for the intake, exhaust, EGR and intermediate manifold, respectively, *c* for the compressor, *t* for the turbine, "*egr,1*" for the control valve 1, "*egr,2*" for the automatic valve 2, *f* for the fuel. Double subscripts denote the mass flow direction, with the first being the upstream and the second being downstream location. The mass flows are defined as follows

$$W_{egr,2}(t) = A_{egr,2}(u_{egr,2}(t))\frac{p_4(t)}{\sqrt{R_x T_4}}\Psi\left(\frac{p_i(t)}{p_4(t)}\right) \tag{2a}$$

$$W_{ie}(t) = \frac{V_d \omega_e(t) p_i(t)}{v 2\pi R_i T_i}\eta_v(\omega_e(t), p_i(t), \dots) \tag{2b}$$

$$W_{exh}(t) = A_{exh}(u_{exh}(t))\frac{p_x(t)}{\sqrt{R_x T_x}}\Psi\left(\frac{p_a}{p_x(t)}\right) \tag{2c}$$

$$W_{egr,1}(t) = A_{egr,1}(u_{egr,1}(t))\frac{p_x(t)}{\sqrt{R_x T_x}}\Psi\left(\frac{p_{egr}(t)}{p_x(t)}\right) \tag{2d}$$

$$W_{vgt}(t) = A_{vgt}(u_{egr}(t))\frac{p_{egr}(t)}{\sqrt{R_x T_3}}\Psi\left(\frac{p_a}{p_{egr}(t)}\right) \tag{2e}$$

$$W_{ci}(t) = \frac{\eta_c P_c(t)}{c_{p,c} T_a\left(\left(\frac{p_i(t)}{p_a}\right)^{\mu_c} - 1\right)} \tag{2f}$$

$$W_{comp}(t) = f\left(\omega_{tc}(t), \frac{p_4(t)}{p_{egr}(t)}\right), \tag{2g}$$

where $\eta_c$ is the compressor efficiency, $T_a$ and $p_a$ are the ambient temperature and pressure, $c_{p,i}$ is the gas specific heat capacity in constant pressure, $\mu_c = \gamma_c / (\gamma_c - 1)$ and $\gamma_c = c_{p,i}/c_{v,i}$, $A$ is the effective area of the valve, $u$ is the control signal, $V_d$ is the engine displacement volume, $\omega_e$ its angular speed, $v$ is the number of revolutions per cycle (2 for 4-stroke engine), $\eta_v(\dots)$ is the engine volumetric efficiency, $\psi(p_r)$ is the pressure ratio correction factor.

$$\Psi(p_r) = \begin{cases} \sqrt{\dfrac{2\gamma}{\gamma - 1}\left(p_r^{\frac{2}{\gamma}} - p_r^{\frac{\gamma+1}{\gamma}}\right)}, & if\ p_r > r_c \\ \gamma^{\frac{1}{2}}\left(\dfrac{2}{\gamma + 1}\right)^{\frac{\gamma+1}{2(\gamma-1)}}, & if\ p_r \le r_c \end{cases} \tag{3}$$

where the pressure ratio is $p_r = p_{ds}(\mathrm{t}) / p_{us}(\mathrm{t})$, critical pressure ratio is $r_c = \left(\frac{2}{\gamma_e+1}\right)^{\frac{\gamma_e}{\gamma_e-1}}$ and $\gamma_e$ is the ratio of the gas specific heats $c_p$ and $c_v$ at constant pressure and at constant volume, respectively. The function for turbo-compressor mass flow $W_{comp}(t)$ calculation is implemented as a 2-D lookup table based on the available compressor map. The turbine power $P_t(t)$ can be calculated as

$$P_c(t) = W_{vgt}(t) c_p T_3 \left(1 - \left(\frac{p_a}{p_{egr}(t)}\right)^{\mu_c}\right) \tag{4}$$

Assuming perfect gas mixing, the exhaust gas fraction (denoted $\chi$ in this article) recirculated to the intake manifold can be calculated as a ratio between the exhaust gas flow and a total mass available in the intake

$$\chi(t) = \frac{W_{egr,2}(t)}{W_{egr,2}(t) + W_{ci}(t)} 100\% \tag{5}$$

Combining Eq. (1a)-(1f), (2a)-(2g), (3)-(5), the non-linear discrete-time system can be written:

$$\begin{aligned} x(k+1) &= f\big(x(k), u(k)\big) + w(k) \\ y(k) &= g\big(x(k)\big) + v(k) \end{aligned} \tag{6}$$

where the state vector $x \in R^n$, the input vector $u \in R^m$ and the measurement vector y $\in R^m$ are defined as

$$\begin{aligned} x &= [p_i \quad p_x \quad p_{egr} \quad p_4 \quad \omega_{tc} \quad P_c]^T \\ u &= [u_{egr,1} \quad u_{vgt}]^T \\ y &= [p_i \quad \omega_{tc}]^T \end{aligned} \tag{7}$$

respectively. The process and measurement models are $f\big(x(k), u(k)\big): R \to R^n$ and $\mathrm{g}\big(x(k)\big): R \to R^n$, respectively; $w(k) \sim N(0, Q(k))$ is the process noise and $\mathrm{v}(k) \sim N(0, R(k))$ is the measurement noise.

# 3 CONTROL DESIGN

## 3.1 Model predictive controller

In this section, we design a non-linear input-constrained model predictive controller. We note that, MPC is an optimal control algorithm capable of dealing with multi-input multi-output systems and explicitly including constraints into design /9/. These factors make it especially attractive for our application.

The main idea of the MPC is to optimize the plant output based on the predictions obtained from the models within a certain horizon $N$ (Fig. 2, left). The analysis of the first principles non-linear engine model has shown that it could be adequately described by three linear models. Therefore, three controllers have to be developed to cover the engine operating range and a switching mechanism is implemented as a function of the EGR reference value.

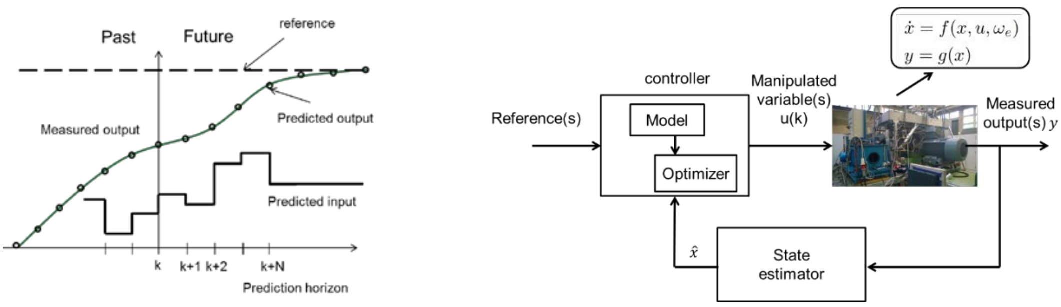


**Figure 2.** General concept of MPC (left) and engine model predictive control configuration (right).

The MPC configuration is depicted in Fig. 2,right and the design is summarized as follows:

1. **Linear model of the system.** The linear state-space models are obtained by using the Simulink input/output linearization tools.
2. **Prediction model.** The N-steps ahead prediction models are formed from the state-space models as follows

$$\begin{bmatrix} x_{k+1|k} \\ x_{k+2|k} \\ \vdots \\ x_{k+N|k} \end{bmatrix} = \begin{bmatrix} A \\ A^2 \\ \vdots \\ A^N \end{bmatrix} x_k + \begin{bmatrix} B & 0 & \cdots & 0 \\ AB & B & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ A^{N-1}B & A^{N-2}B & \cdots & B \end{bmatrix} \begin{bmatrix} u_{k|k} \\ u_{k+1|k} \\ \vdots \\ u_{k+N-1|k} \end{bmatrix}$$
$$\begin{bmatrix} y_{k+1|k} \\ y_{k+2|k} \\ \vdots \\ y_{k+N|k} \end{bmatrix} = \begin{bmatrix} CA \\ CA^2 \\ \vdots \\ CA^N \end{bmatrix} x_k + \begin{bmatrix} CB & 0 & \cdots & 0 \\ CAB & CB & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ CA^{N-1}B & CA^{N-2}B & \cdots & CB \end{bmatrix} \begin{bmatrix} u_{k|k} \\ u_{k+1|k} \\ \vdots \\ u_{k+N-1|k} \end{bmatrix} \tag{8}$$

where $A \in R^{n\times n}$, $B \in R^{n\times m}$ and $C \in R^{m\times n}$ are the state, input and output matrices and *N* is the prediction horizon.

3. **State estimator.** Full state is required for the model to predict the future output for a specified prediction horizon. In this work a linear Kalman filter is used

   *Update:* $\hat{x}(k|k) = \hat{x}(k|k-1) + M(y_m(k) - \hat{y}_m(k))$ (9)

   *Prediction:* $\hat{x}(k+1|k) = A\hat{x}(k|k) + Bu(k)$ (10)

   $\hat{y}_m(k) = C\hat{x}(k|k-1)$ (11)

   where *M* is the optimal gain.
4. **Cost function.** Includes two terms: tracking error $e_k = r_k - y_k$ penalty and control deviation penalty

   $J = \sum_{k=1}^{n_y} \left\| W_y e_k \right\|^2{}_2 + \sum_{k=1}^{n_u} \| W_u (u_k - u_{ss}) \|^2{}_2$ (12)

   where $W_y$ and $W_u$ are the tracking error and input weights, $n_y$ and $n_u$ is the amount of outputs and inputs, respectively and $u_{ss}$ is the control signal stead-state value.
5. **Optimization problem.** A constrained optimization problem, which minimizes the cost function over the whole prediction horizon *N* is defined as follows

   $\min_{u_{k|k} \cdots u_{k+N-1|k}} \sum_{i=0}^{N-1} \left( \sum_{j=1}^{n_y} \left\| W_y \left( y_j(k+i+1|k) - r_j(k+i+1) \right) \right\|^2{}_2 + \sum_{j=1}^{n_u} \left\| W_u \left( u_j(k+i|k) - u_{ss,j}(k+i) \right) \right\|^2{}_2 \right)$ (13)

   s.t. $u_{j,min}(i) < u_j(k+i|k) < u_{j,max}(i)$

## 3.2 H∞ controller

We also designed a mixed-sensitivity $H_\infty$ state-space controller as another advanced control algorithm to validate the advantages of MPC. We note that, the design of $H_\infty$ controller is in general (even with the presence of design toolboxes) more complicated and requires a good insight into systems frequency response.

The control algorithm can be summarized as follows (/13/):

1. Define a stacked requirements as a vector $V = \begin{bmatrix} w_P S \\ w_T T \\ w_u KS \end{bmatrix}$, where *S*, *T* and *KS* are the sensitivity, complimentary sensitivity and the controller sensitivity functions and $w_P, w_T$ and $w_u$ are the corresponding penalizing weights and a maximum singular value $\|V\|_\infty = \max_\omega \bar{\sigma}(V(j\omega)) < 1$ is bounded.
2. Find a stabilizing controller by solving a minimization problem $\min_K \|V(K)\|_\infty$

This yields the controller that shapes the following transfer functions *S*, *KS* and *L* (Fig. 3), where L is the open-loop transfer function. It can be seen that three different controllers yield similar shape for *L*, which should provide a similar response in the closed-loop for the original plant.

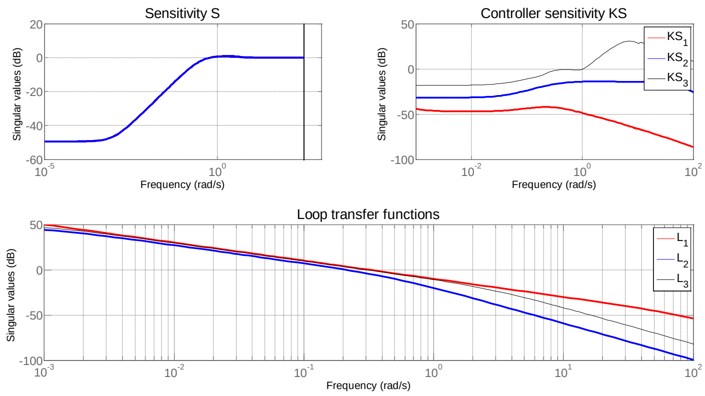

**Figure 3.** Sensitivity $S$, controller sensitivity $KS$ and open-loop $L$ transfer functions for three controllers.

## 4 NUMERICAL SIMULATION

The designed controllers are applied to the original nonlinear model of the engine and their performance is compared. For the sake of comparison and due to a large magnitude of the controlled variables, we normalize them as follows:

- $\chi_{norm} = \frac{\chi_{set-point}}{\chi}$
- $\omega_{norm} = \frac{\omega_{set-point}}{\omega_{tc}}$

The normalized plots for the whole EGR range (14-32%, with 2% step) is shown in Fig. 4, left. The turbocharger speed $\omega_{tc}$ is kept constant. We note that the EGR valve control is a servo-problem and VGT is a disturbance rejection. The controller-switching signal has three states: *one* (0 – 1000 sec), *two* (1000 – 1700 sec) and *three* (1700 – 2500 sec) to select a certain controller. It can be seen that a model accuracy affects the controller performance a lot. For instance, the MPC performance is the best in state *one* and *two*, but degrades in region *three*. Since we use the same models for both controllers, the same degradation happens with the $H_\infty$ controller.

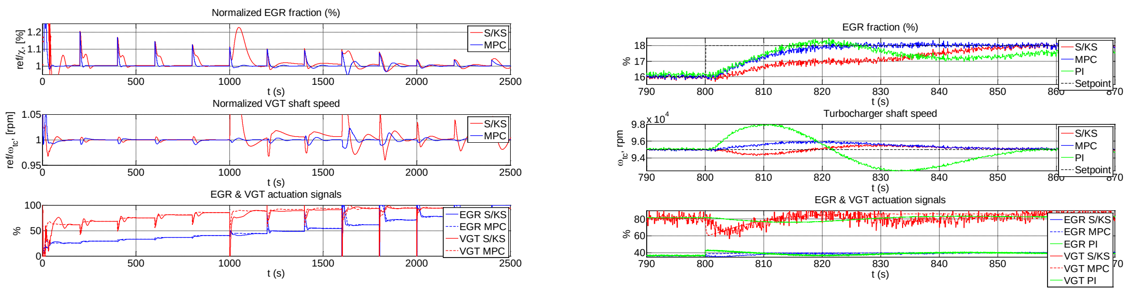

**Figure 4.** Normalized EGR and $\omega_{tc}$. Multivariable MPC and $H_\infty$ control comparison for EGR (top) and $\omega_{tc}$ (middle) tracking. Control signals for EGR valve and VGT are also shown (low).

Since the $H_\infty$ controllers are known to have a problem of amplifying the feedback noise due to their derivative action, we also simulate the plants response with the measurement noise included. The step response of the EGR fraction and the $\omega_{tc}$ behaviour for the designed controllers as well as for the PI controller are shown in Fig. 4, right. The advantage of the multivariable control strategy is clearly seen in comparison to the decentralized PI controllers. Also the noise amplification by the $H_\infty$ is evident and the weights for $S$ transfer function should be carefully designed, to mitigate the noise.

## 5 CONCLUSION

The problem of delivering high and stable portion of EGR over the engine operating range is addressed in this work. A novel turbocharged HPEGR system is proposed to tackle this problem. In authors opinion this type of an EGR structure has not been researched enough and it is therefore important to investigate it.

In this work, a generic mean-value modelling algorithm of HPEGR system has been presented and a multivariable model predictive controller has been designed. MPC is a modern tool and is relatively new to the marine industry. Therefore, the main task was to evaluate its performance for the HPEGR application. MPC is an optimal control and can provide a superior performance, taking into account optimization constraints (states, output and inputs). MPC structure also allows handling of multivariable systems, which is a definite advantage over decentralized PI control algorithms. During simulations, MPC has proved the best in terms of rise time, settling and disturbance rejection.

However, its design can be complicated, as it requires a bunch of models (three in EVE's case), or online sequential linearization to adequately represent the engine behaviour.

Experimental verification of the model has been done with the data obtained in a laboratory engine test-bed. However, this engine is not a production type and is not suitable for dynamic testing, which is the main issue in control implementation.

Further work should include a full-scale laboratory tests to verify the designed estimation and control algorithms.

## 6 ACKNOWLEDGMENTS

This research has received funding from the HERCULES-C project, funded by the European Commission, DG Research, under Contract SCP1-GA-2011-284354.

## REFERENCES

[1] Codan, E., Horler, H., Stebler, H. and Widenhorn, M. "Method and apparatus for high-pressure end exhaust gas recirculation on a supercharged internal combustion engine." U.S. Patent 5,564,275, issued October 15, 1996.
[2] Grondin, O., Moulin, P. and Chauvin, J. "Control of a turbocharged Diesel engine fitted with high pressure and low pressure exhaust gas recirculation systems." In *Proceedings of Decision and Control, 2009 held jointly with the 2009 28th Chinese Control Conference. CDC/CCC 2009*, pp. 6582-6589.
[3] Haber, B. A Robust Control Approach on Diesel Engines with Dual-Loop Exhaust Gas Recirculation Systems. Master's thesis, the Ohio State University, 2010.
[4] Heywood, J. *Internal combustion engine fundamentals*. Vol. 930. New York: Mcgraw-hill, 1988.
[5] Jankovic, M. and Kolmanovsky, I. "Robust nonlinear controller for turbocharged diesel engines." In *Proceedings of the American Control Conference, 1998.*, vol. 3, pp. 1389-1394.
[6] Jung, M. "Mean-value modelling and robust control of the airpath of a turbocharged diesel engine." PhD diss., University of Cambridge, 2003.
[7] Jung, M, Ford, R., Glover, K., Collings, N., Christen, U. and Watts, M. *Parameterization and transient validation of a variable geometry turbocharger for mean-value modeling at low and medium speed-load points*. No. 2002-01-2729. SAE Technical Paper, 2002.
[8] Kolmanovsky, I., Morall, P., Nieuwstadt, M. and Stefanopoulou, A. "Issues in modelling and control of intake flow in variable geometry turbocharged engines." *Chapman and Hall CRC research notes in mathematics*(1999): 436-445.
[9] Maciejowski, J. *Predictive control: with constraints*. Pearson education, 2002.
[10] Reifarth, S. "EGR-Systems for Diesel Engines." KTH Royal Inst. of Technology, Stockholm (Sweden). School of Industrial Engineering and Management, Machine Design, 2010.
[11] Samokhin, S., Sarjovaara, T., Zenger, K. and Larmi M. "Modeling and Control of Diesel Engines with a High-Pressure Exhaust Gas Recirculation System." In *Proceedings of the 19th IFAC World Congress*, vol. 19, no. 1, pp. 3006-3011. 2014.
[12] Shao, J., Hwang, L., Miller, P. and Charlton, S. "EGR delivery and control system using dedicated full authority compressor." U.S. Patent 6,216,461, issued April 17, 2001.
[13] Skogestad, S. and Postlethwaite, I. *Multivariable feedback control: analysis and design*. Vol. 2. New York: Wiley, 2007.
[14] Stefanopoulou, A., Kolmanovsky, I., and Freudenberg, J. "Control of variable geometry turbocharged diesel engines for reduced emissions." *IEEE Transactions on Control Systems Technology, vol.* 8, no. 4, pp. 733-745, 2000.
[15] Wahlström, J., and Eriksson, L. "Modelling diesel engines with a variable-geometry turbocharger and exhaust gas recirculation by optimization of model parameters for capturing non-linear system dynamics." *Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering* 225, no. 7 (2011): 960-986.
[16] Wang, H., Bosche, J., Tian, Y., and Hajjaji, A.. "Two loop based dynamical feedback stabilization control of a diesel engine with EGR & VGT." In *Proceedings Decision and Control and European Control Conference (CDC-ECC), 2011 50th IEEE Conference on*, pp. 1596-1601. IEEE, 2011.