Robust receding horizon control using tubes – an application to operational optimization of water distribution systems with uncertain water demand

István SELEK,
Systems Engineering Laboratory, Oulu, Finland
POB 4300 FIN-90014 University of Oulu
istvan.selek@oulu.fi, tel.: +358 50 350 6837

Abstract

In this paper an approach is proposed for the robust optimal control of water distribution systems subject to water demand uncertainties. The underlying mathematical problem is formulated as an optimal target tube reachability problem where an approximate policy is computed utilizing permutational symmetries and receding horizon principle. An application for the robust optimal control of the waterworks of Sopton (Hungary) is presented.

Key words: Optimal control, Robust control, Model predictive control

1 Introduction

Operational optimization of water distribution systems is a practical problem with high importance. It aims at satisfying residential and industrial water demand while minimizing the corresponding operational costs under various constraints mainly related to safety and storage capacity. In water supply networks cost optimal operation is achieved by a management policy implemented as operation rules for the active hydraulic elements (pumps, valves) of the network. Policy is to be based on all available information on the system's state, to compensate for the disturbances of the system which originates from consumer behavior. The problem of designing optimal management policy for water supply systems is highly challenging, involving (high) dimensionality, nonlinearity and uncertainty in water demand.

Typically, human operators use heuristic ideas and/or rules of thumb to minimize operational costs. On the other hand, the field has been attracting numerous researchers resulting in a various techniques and proposals for minimizing operative costs associated with pumping systems of water supply. However, as of 2015, the mathematical formulation of the problem heavily relies on deterministic assumptions in the vast majority of the cases, while (despite the stochastic nature of the underlying problem) little attention has been paid for formulating the problem in the presence of uncertainly of consumer behavior. This results in the well-known fact that, the vast majority of the proposals address nonlinearity and dimensionality issues while the uncertainty of water demand is almost completely ignored in the corresponding literature. The approaches are usually centered around the application of linear (Giacomello et al., 2013) and nonlinear programming techniques (Cembrano et al., 2004), while recently metaheuristics (Georgescu and Georgescu, 2014) (e.g. Genetic Algorithms, Evolutionary Strategies, Neural Networks etc.) gained much interest.

In this paper a robust approach is proposed for the optimal control of water distribution systems subject to water demand uncertainties. The approach exploits the characteristics of a "well designed" water network which can be summarized as follows:

- Internal network pressure remains within acceptable bounds for allowable service reservoir storage fluctuations;
- The head lift for each pumping station is large compared to the network nodal head changes induced by pump/valve switchings elsewhere in the system;
- The flows a given pumping station will deliver depend on zonal consumer demands, and not on the changes in the network head/flow pattern caused by pump/valve switchings elsewhere in the system.

Based on these assumptions a management policy is calculated on–line utilizing the receding horizon principle. The obtained policy is optimality and feasibility robust, that is, it remains nearly optimal and feasible over all admissible realizations of the water demand scenario. An application of the proposed approach for the least cost energy control of the water system of Sopron (Hungary) is presented.

2 Problem statement

Utilizing the outlined characteristics of a well designed water networks, the dynamic behavior of the network can be well approximated by linear (aggregated) mass balance models. An aggregated model of a water distribution system has the following form

$$\mathbf{x}(k+1) = \mathbf{x}(k) + \mathbf{B}\mathbf{u}(k) + \mathbf{D}\mathbf{w}(k), \quad k = 0, \ldots, K-1 \tag{1}$$

where

- $\mathbf{x}(k) \in \bar{\mathcal{X}}_k \subseteq \mathcal{R}^n$ (m^3) denotes the amount of stored water in tanks at time $t = k\Delta t$, where $t \in \mathcal{R}_+$ and $\Delta t > 0$ (h) is the sampling time,
- $\mathbf{u}(k) \in \mathcal{U}_k \subseteq \mathcal{R}^m$ (m^3) is the amount of water flows through the hydraulic actuators (pumps, valves) on time horizon $[k\Delta t, (k+1)\Delta t)$,
- $\mathbf{w}(k) \in \mathcal{W}_k \subseteq \mathcal{R}^z$ (m^3) is the water demand, quantifying the consumed water by users on time horizon $[k\Delta t, (k+1)\Delta t)$,
- $\mathbf{B} \in \{-1, 0, 1\}^{n \times m}$ and $\mathbf{D} \in \{-1, 0, 1\}^{n \times z}$ are system matrices describing the topology of the water network.

Furthermore, at each time instant k a cost $c(\mathbf{u}(k), k)$ (referred to as step cost) occurs and accumulates over time. Using these, the following assumptions are taken:

1. The sets $\bar{\mathcal{X}}_k$ (referred to as effective target sets), are measurable on $\mathcal{R}^n$ and characterized by a polyhedral set of the form
$$\bar{\mathcal{X}}_k = \left\{\mathbf{x} \in \mathcal{R}^n \mid \bar{\mathbf{b}}_k \leq \bar{\mathbf{A}}_k \mathbf{x}\right\}; \tag{2}$$
2. The control domains $\mathcal{U}_k$ are measurable on $\mathcal{R}^m$ and characterized by a polyhedral set of the form
$$\mathcal{U}_k = \left\{\mathbf{u} \in \mathcal{R}^m \mid \mathbf{b}_{\mathrm{u},k} \leq \mathbf{A}_{\mathrm{u},k}\mathbf{u}\right\}; \tag{3}$$
3. The sets $\mathcal{W}_k$ are convex and compact;

4. The step cost $c(\mathbf{u}(k), k)$ is spatially separable, that is

$$c(\mathbf{u}(k), k) = \sum_{i=1}^{m} c_i(u_i(k), k) \tag{4}$$

and $c_i(u_i(k), k)$ are convex or can be well approximated by convex functions.

2.1 The optimal target tube reachability problem

Relying on the outlined concepts and assumptions the optimal control problem is formulated as follows: find a sequence of functions (policy) $\pi^* = \{\boldsymbol{\pi}_0, \dots, \boldsymbol{\pi}_{K-1}\}$ ($\boldsymbol{\pi}_k : \bar{\mathcal{X}}_k \to \mathcal{U}_k$) which map the states into feasible controls at each time instant k such that $\mathbf{u}(k) = \boldsymbol{\pi}_k(\mathbf{x}(k))$ and maintain the state trajectory $\{\mathbf{x}(1), \dots, \mathbf{x}(K)\}$ within the effective target tube

$$\{\bar{\mathcal{X}}_1, \bar{\mathcal{X}}_2, \dots, \bar{\mathcal{X}}_K\} \tag{5}$$

while minimizing the associated cost

$$J(\pi) = \lim_{K\to\infty} \frac{1}{K} \left(\underset{\mathbf{w}(0),\dots,\mathbf{w}(K-1)}{E} \left\{ \sum_{k=0}^{K-1} c\big(\boldsymbol{\pi}_k(\mathbf{x}(k)),\, k\big) \right\} \right), \tag{6}$$

subject to (1 – 5).

3 Problem solution

This section proposes an approach which provides an approximate solution to the outlined optimal control problem utilizing the receding horizon principle and a system property called *permutational invariance* which refers to the invariance of the state under under input control sequence permutations Bene and Selek (2012); Selek et al. (2013). The key idea is as follows: utilizing permutational invariance a pseudo state space and a corresponding deterministic dynamic model is constructed. Then the target tube reachability problem is reformulated and solved (by means of deterministic optimization techniques) using the pseudo model and the obtained control decision is implemented for the original system as an approximate policy. Reformulating (1) we get

$$\mathbf{x}(k+i+1) = \mathbf{x}(k) + \mathbf{B}\boldsymbol{\xi}(i+1) + \mathbf{D}\sum_{j=0}^{i} \mathbf{w}(k+j). \tag{7}$$

where

$$\boldsymbol{\xi}(i+1) = \boldsymbol{\xi}(i) + \mathbf{u}(k+i), \quad i = 0, 1, \dots \tag{8}$$

specifies the pseudo dynamics and $\boldsymbol{\xi}$ is the pseudo state of the system. Using (8) the optimal target tube reachability problem is reformulated in the pseudo state space and solved on a rolling horizon $i = 1, \dots, N$. Condition (5) requires that the pseudo state trajectory $\{\boldsymbol{\xi}(1), \dots, \boldsymbol{\xi}(N)\}$ must stay within the effective target tube $\left\{\bar{\mathcal{E}}_1, \dots, \bar{\mathcal{E}}_N\right\}$ defined in pseudo state space. The calculation of the effective target tube begins with the characterization of target sets $\mathcal{E}_{i+1}$ in the pseudo state space. It is easy to see that, if $\mathbf{u}(k+i) \in \mathcal{U}_{k+i}$ then the pseudo state $\boldsymbol{\xi}(i+1)$ must stay within

$$\mathcal{E}_{i+1} = \bigoplus_{j=0}^{i} \mathcal{U}_{k+j}, \tag{9}$$

where $\bigoplus$ denotes the Minkowski sum of sets. Due to the fact that the control domains $\mathcal{U}_{k+j}$ are polyhedral the target sets in pseudo state space are defined by linear inequalities as follows

$$\mathcal{E}_{i+1} = \left\{\boldsymbol{\xi} \in \mathcal{E} \,|\, \boldsymbol{\beta}_{i+1} \leq \boldsymbol{\Lambda}_{i+1}\boldsymbol{\xi}\right\}, \quad i = 0, 1, \ldots, N-1. \tag{10}$$

On the other hand, condition (5) yields

$$\bar{\mathbf{b}}_{k+i+1} \leq \bar{\mathbf{A}}_{k+i+1}\left(\mathbf{x}(k) + \mathbf{B}\boldsymbol{\xi}(i+1) + \mathbf{D}\sum_{j=0}^{i}\mathbf{w}(k+j)\right), \quad i = 0, 1, \ldots, N-1. \tag{11}$$

Putting these together, the inequality

$$\begin{pmatrix} \bar{\mathbf{b}}_{k+i+1} - \bar{\mathbf{A}}_{k+i+1}\mathbf{x}(k) - \bar{\mathbf{A}}_{k+i+1}\mathbf{D}\sum_{j=0}^{i}\mathbf{w}(k+j) \\ \boldsymbol{\beta}_{i+1} \end{pmatrix} \leq \begin{pmatrix} \bar{\mathbf{A}}_{k+i+1}\mathbf{B} \\ \boldsymbol{\Lambda}_{i+1} \end{pmatrix} \boldsymbol{\xi}(i+1) \tag{12}$$

must be satisfied at each time instant i for all possible realization of the corresponding disturbance trajectory. To acieve this a vector $\boldsymbol{\varpi}_{i+1}$ is calculated where

$$\boldsymbol{\varpi}_{i+1} = \sup_{(\mathbf{w}(k),\ldots,\mathbf{w}(k+i)) \in (\mathcal{W}_k \times \cdots \times \mathcal{W}_{k+i})} \bar{\mathbf{b}}_{k+i+1} - \bar{\mathbf{A}}_{k+i+1}\mathbf{x}(k) - \bar{\mathbf{A}}_{k+i+1}\mathbf{D}\sum_{j=0}^{i}\mathbf{w}(k+j). \tag{13}$$

The optimization is carried out over the domain $\mathcal{W}_k \times \cdots \times \mathcal{W}_{k+i}$ on each coordinate independently. The vector of upper bounds $\boldsymbol{\varpi}_{i+1}$ defines the robust condition, that is, the inequality

$$\begin{pmatrix} \bar{\mathbf{b}}_{k+i+1} - \bar{\mathbf{A}}_{k+i+1}\mathbf{x}(k) - \bar{\mathbf{A}}_{k+i+1}\mathbf{D}\sum_{j=0}^{i}\mathbf{w}(k+j) \\ \boldsymbol{\beta}_{i+1} \end{pmatrix} \leq \begin{pmatrix} \boldsymbol{\varpi}_{i+1} \\ \boldsymbol{\beta}_{i+1} \end{pmatrix} \tag{14}$$

is satisfied at any time instant i for any admissible realization of the corresponding disturbance trajectory. Finally the effective target set

$$\bar{\mathcal{E}}_{i+1} = \left\{\boldsymbol{\xi} \in \mathcal{E} \,|\, \bar{\boldsymbol{\beta}}_{i+1} \leq \bar{\boldsymbol{\Lambda}}_{i+1}\boldsymbol{\xi}\right\} \tag{15}$$

in the pseudo state space is specified, where

$$\bar{\boldsymbol{\beta}}_{i+1} = \begin{pmatrix} \boldsymbol{\varpi}_{i+1} \\ \boldsymbol{\beta}_{i+1} \end{pmatrix}, \quad \bar{\boldsymbol{\Lambda}}_{i+1} = \begin{pmatrix} \bar{\mathbf{A}}_{k+i+1}\mathbf{B} \\ \boldsymbol{\Lambda}_{i+1} \end{pmatrix}. \tag{16}$$

Using these the optimal control problem is formulated as follows: find a control sequence $\{\mathbf{u}(k), \mathbf{u}(k+1), \ldots, \mathbf{u}(k+N-1)\}$ $(\mathbf{u}(k+i) \in \mathcal{U}_{k+i})$ which maintains the pseudo state trajectory $\{\boldsymbol{\xi}(1), \boldsymbol{\xi}(2), \ldots, \boldsymbol{\xi}(N)\}$ within the effective target tube $\{\bar{\mathcal{E}}_1, \bar{\mathcal{E}}_2, \ldots, \bar{\mathcal{E}}_N\}$ and minimizes the associated cost $\sum_{i=1}^{N} c(\mathbf{u}(k+i), k+i)$ subject to dynamics (8). The solution to the defined optimal control problem can be obtained by means of deterministic convex programming.

4 Application

The proposed approach is applied for the robust optimal control of the waterworks of Sopron (Hungary). The topology of the water distribution system is shown in Figure 1. The network consists of 11 pumping stations, 8 reservoirs and 5 main water demand zones allocated to the corresponding service reservoirs.

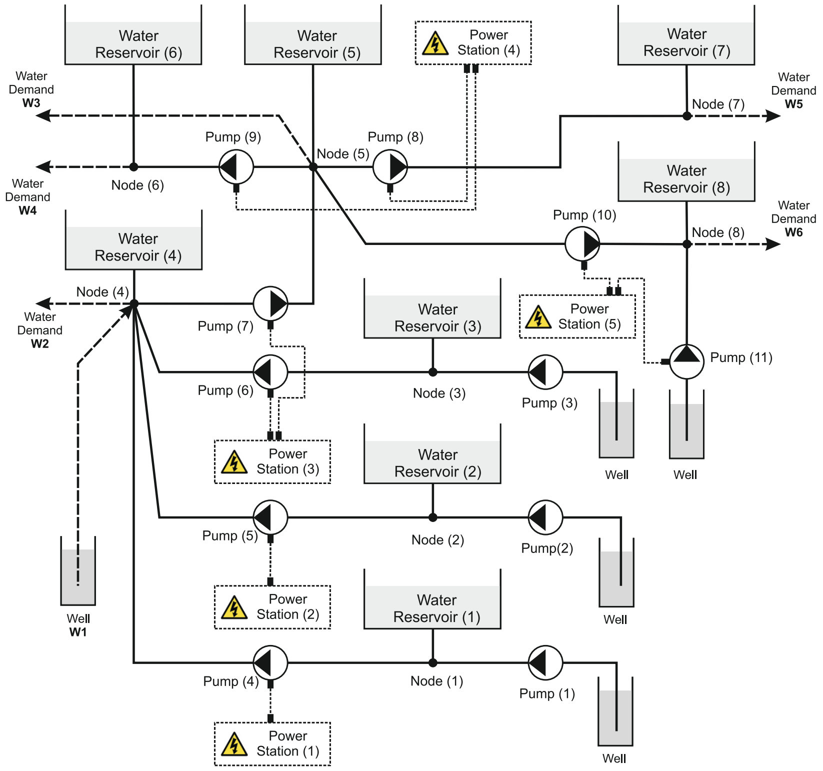

Figure 1: The topology of the regional water distribution network of Sopron, Hungary

The goal is to minimize the total cost of water transport i.e. the cost of electric energy consumed by pumps while satisfying the water demand subject to reservoir constraints. The cost of electric energy consumed by pump j at time k is

$$c_j(u_j(k), k) = P_j(u_j(k), k)\varepsilon(k) \tag{17}$$

where P_j is the power consumption of the corresponding pumping station and $\varepsilon(k)$ denotes the price of the electric energy. The energy tariff varies during the day, involving peak (price of electric energy is high) and off peak periods (price of electric energy is low). The tariff has the following pattern: 1 (Unit) $\{[0\text{h} - 7\text{h}), [13\text{h} - 17\text{h}), [20\text{h} - 24\text{h})\}$ and 1.25 (Unit) $\{[7\text{h} - 13\text{h}), [17\text{h} - 20\text{h})\}$. Unit denotes the price of the electric energy in terms of a given currency (e.g. EUR/kWh, USD/kWh etc.). The water demand is generated according to uniform distribution

$$w_j(k) \sim U\big(w_j^{\min}(k), w_j^{\max}(k)\big), \quad j = 1, \dots, 6 \tag{18}$$

where the distribution parameters $(w_j^{\min}(k), w_j^{\max}(k))$ were obtained using historical records. The parameters were considered as periodic with a period of one year. The historical records show that the consumption uncertainty $0.5(w_j^{\max}(k) - w_j^{\min}(k))$ varies between $5 - 50\%$ of the nominal water consumption. Figure 2 shows the optimal control strategy and the evolution of reservoir storage for 10 days of operation using a random feasible state as initial condition. The corresponding optimal cost per stage was 289.25 units.

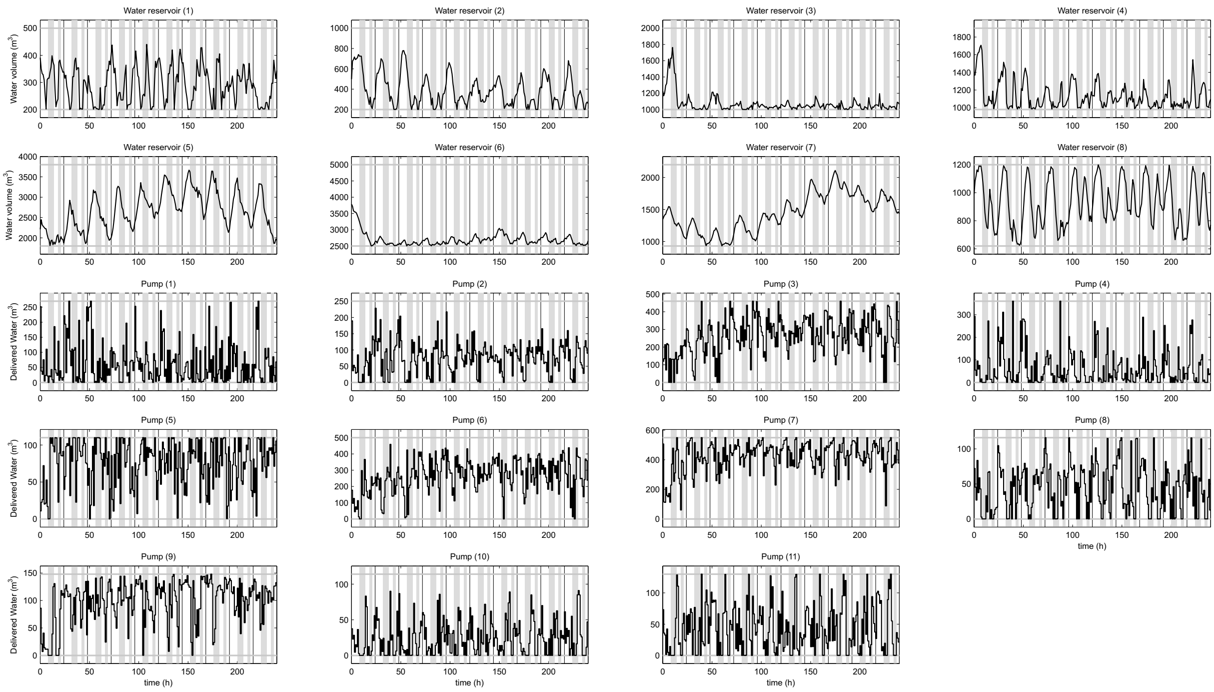

Figure 2: 10 day optimal pump control policy. Peak charging periods are gray shaded while off-peak periods are uncolored. Reservoir and pump delivery upper and lower bounds are indicated by solid gray lines.

5 Summary and conclusions

In this paper an approach was proposed for the robust optimal control of water distribution systems subject to water demand uncertainties. The main characteristics of a "well" designed water network was utilized to formulate an aggregate mathematical model and the corresponding optimal target tube reachability problem. The optimal solution to the defined problem is approximated using permutational symmetries and the receding horizon principle. The approximate policy is feasibility and optimality robust (it remains feasible and (sub) optimal for all admissible realizations of the water demand.) An application for the least cost pump scheduling of the waterworks of Sopron was included. Although the presented results are preliminary, these indicate a great potential of the presented approach.

References

Bene, J. G. and I. Selek (2012). Water network operational optimization: Utilizing symmetries in combinatorial problems by dynamic programming. *Periodica Polytechnica, Civil Engineering 56*(1), 51–61.

Cembrano, G., J. Quevedo, M. Salamero, V. Puig, J. Figueras, and J. Marti (2004). Optimal control of urban drainage systems. a case study. *Control Engineering Practice 12*(1), 1–9.

Georgescu, S. C. and A. M. Georgescu (2014). Application of HBMOA to pumping stations scheduling for a water distribution network with multiple tanks. *Procedia Engineering 70*, 715–723.

Giacomello, C., Z. Kapelan, and M. Nicolini (2013). Fast hybrid optimization method for effective pump scheduling. *Journal of Water Resources Planning and Management 139*(2), 175–183.

Selek, I., J. G. Bene, and E. Ikonen (2013). Utilizing permutational symmetries in Dynamic Programming - with an application to the optimal control of water distribution systems under water demand uncertainties. *International Journal of Innovative Computing, Information and Control 9*(8), 3091–3113.