Composite Nonlinear Feedback Control of a Chemical Reactor

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ABSTRACT

This paper studies the application of composite nonlinear feedback (CNF) control for a continuous time stirred tank reactor. Inside the reactor, an exothermic chemical reaction occurs, which requires cooling when concentration is commanded from low to high conversion rate to prevent a thermal runaway. A full-state CNF controller is designed for adjusting the temperature of the cooling jacket using concentration and temperature measurements. A continuous time gain-scheduled cascade controller, as well as a model predictive controller (MPC) is also fabricated for comparison. The gain-scheduled cascade controller has a proportional-integral (PI) controller as a primary loop controller, and a P-controller as a secondary loop controller. The simulation results show that the CNF controller is able to offer the best overall tracking performance as measured by the integral-of-absolute-error (IAE) criterion. In addition, the CNF controller does not need gain-scheduling for tuning purposes; the CNF controller is capable of changing its tuning as a function of control error only.

1 INTRODUCTION

A composite nonlinear feedback (CNF) control has been developed to satisfy simultaneous requirements on command following and robustness using constrained control. Generally, the CNF consists of mutually collaborating linear and nonlinear parts, which are designed as follows. First, a linear part is designed to meet sufficient transient speed. Then, the nonlinear part is added to the control law in order to provide control error dependent damping when the output is reaching the desired reference. By such mechanism, the nonlinear part is able to smoothly change the locations of closed-loop poles without any switching elements in the control structure. Therefore, the CNF alters the closed-loop dynamics as desired, which effectively improves the tracking performance.

The CNF has originally been developed to improve the tracking performance of servo systems. However, the potential of the CNF has been recognized in other areas as well. The CNF has been applied; for example, to unmanned aerial vehicles /1/, industrial robotics /5/, hard disk drives /2/, DC-servo motors used in manufacturing industry /4/, amongst others. Is should be to noted that the CNF is highly effective in regulatory control as well. For example, if the system deviates from its current operating point due to some external disturbance, then the tuning of the CNF automatically becomes more aggressive. Such feature helps the system to recover from an unexpected situation in an efficient way.

An exothermic reaction, which requires cooling when the reaction is transferred from low to high conversion rate, is investigated in this paper. The reaction dynamics are nonlinear, which complicates the controller design. Moreover, during the transition, the reaction goes through stable-unstable-stable chain, while the conversion rate increases. Such features usually require gain-scheduling or model predictive control (MPC) in order to control the reaction temperature as the process dynamics evolve.

This paper studies the possibility of CNF for controlling the reactor temperature during the transition. As the concentration and temperature are assumed to be measurable, a full-state CNF controller is designed for adjusting the coolant temperature. A traditional gain-scheduled cascade structure and an MPC-controller are designed for comparison. Within the cascade, a PI-controller is used as the primary loop controller, whereas the secondary loop uses a P-controller. The tracking performances of all control systems are measured using the integral-of-absolute-error (IAE) criterion.

The material of this paper is organized as follows. In Section 2, the reactor is shortly introduced along with the control environment. In Section 3, gain-scheduled cascade, MPC, and CNF controllers are presented for adjusting the coolant temperature of the reactor. Finally, in Section 4, some concluding remarks are drawn.

2 EXOTHERMIC CHEMICAL REACTOR

This paper investigates a continuous time stirred tank reactor, where an exothermic chemical reaction occurs. Necessary background and reactor equations are presented in /10/. The reactor is initially in steady state with feed-in concentration c_{in} = 10kmol/m^3, in-flow temperature T_{in} = 298.2K, and coolant temperature T_c = 297.97K, which is the manipulated variable. The maximum rate of change of the coolant temperature is limited to 10 degrees/min. Initially, the reactor temperature T_{out} = 311.26K and the residual concentration c_{out} = 8.57kmol/m^3. A schematic diagram of the reactor is depicted in Figure 1, respectively.

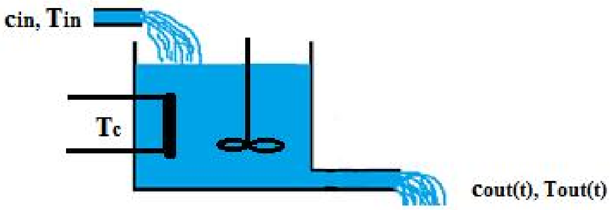

Figure 1. Continuous time stirred tank reactor.

It is assumed that c_{in} and T_{in} remain constant throughout the testing and they cannot be altered by the controller. It is further assumed that the residual concentration and reactor temperature are measured and available for feedback. The flow through the tank remains constant, which retains the level of the reactor. Here, the intention is to transit the reaction from the current state $c_{out}(0)$ = 8.57kmol/m^3 down to $c_{out}(\infty)$ = 2kmol/m^3 linearly in 26 minutes. During the transition, the process change from stable to unstable, and finally back to stable. In the unstable region, appropriate cooling is necessary to prevent thermal runaway. In the following section, three control structures are presented for controlling the reactor temperature during the transition; namely, gain-scheduled cascade control, model predictive control, and composite nonlinear feedback control.

3 CONTROL SYSTEM DESIGN

Gain-Scheduling Cascade Control

Consider the following block diagram in Figure 2, respectively.

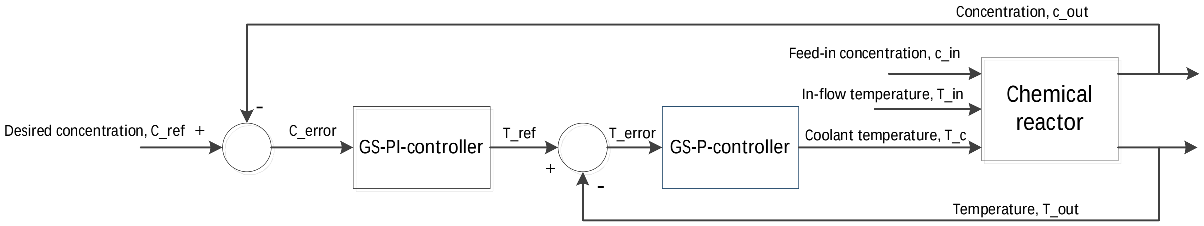

Figure 2. Gain-scheduling cascade structure with primary PI-controller and secondary P-controller.

The structure in Figure 2 represents the well-known cascade control using PI- and P-controllers. However as the reaction display significant dynamic change, gain scheduling is needed in order to perform the control task. Both controllers are tuned using the following quadratic polynomials, which depend on the scheduling variable $c_{out}(t)$

$$\begin{cases} K_p(c_{out}) = K_{p,0} + K_{p,1}c_{out} + K_{p,2}c_{out}^2 \\ K_I(c_{out}) = K_{I,0} + K_{I,1}c_{out} + K_{I,2}c_{out}^2 \\ K_{p2}(c_{out}) = K_{p2,0} + K_{p2,1}c_{out} + K_{p2,2}c_{out}^2 \end{cases}, \tag{1}$$

where K_p represents the primary loop proportional gain, K_I is the primary loop integration gain, and K_{p2} is the secondary loop proportional gain. All other parameters are tunable, which are found based on the actual tuning requirements. It should be noted that the actual tuning change continuously as smoothly as the quadratic polynomials as a function of the residual concentration $c_{out}(t)$ does. The reactor is linearized in 5 different points along the reference trajectory, when the reaction goes through the desired transition.

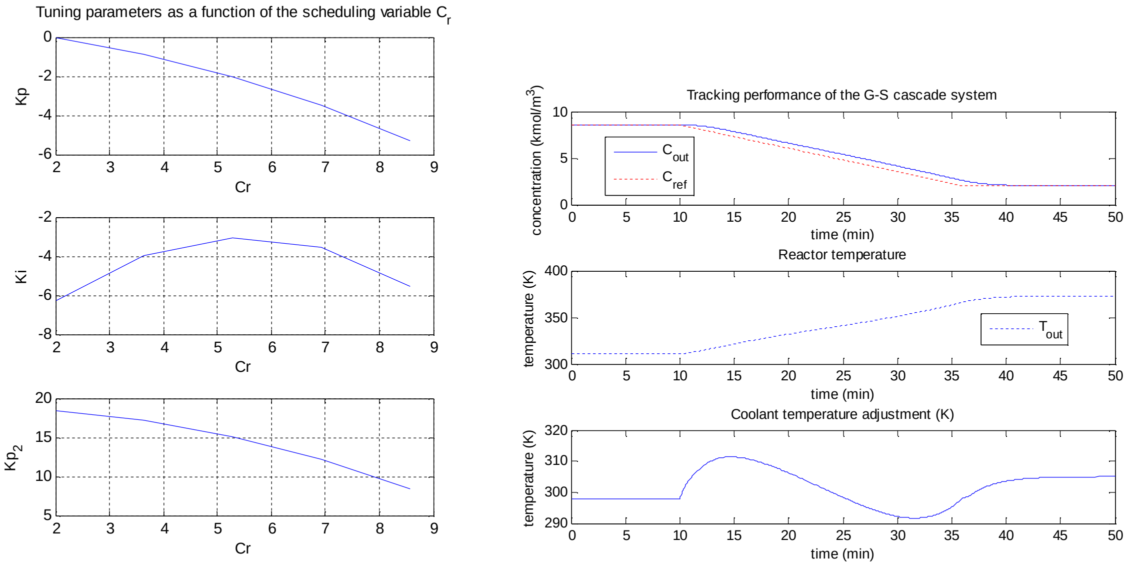

Figure 3. Gain-scheduling tuning and concentration tracking.

Following the procedures given in /8/ results in tuning and tracking performance, which are depicted in the Figure 3, respectively. Judging from the Figure 3, the gains vary substantially within the operation range. The concentration tracking is satisfactory and the coolant temperature change within the given bounds. The IAE = 15.65.

Model Predictive control

Owing to the page limit, only the performance obtained by the MPC controller is presented. The tuning procedure for the MPC can be found in /7/, respectively. It should be noted; however, that it is required to use three different prediction models and three MPC controllers in order to control the coolant temperature satisfactorily throughout the transition. The tracking performance of the MPC is depicted in the Figure 4 with the coolant temperature profile, and a switching signal, which is used to change the prediction model and the MPC.

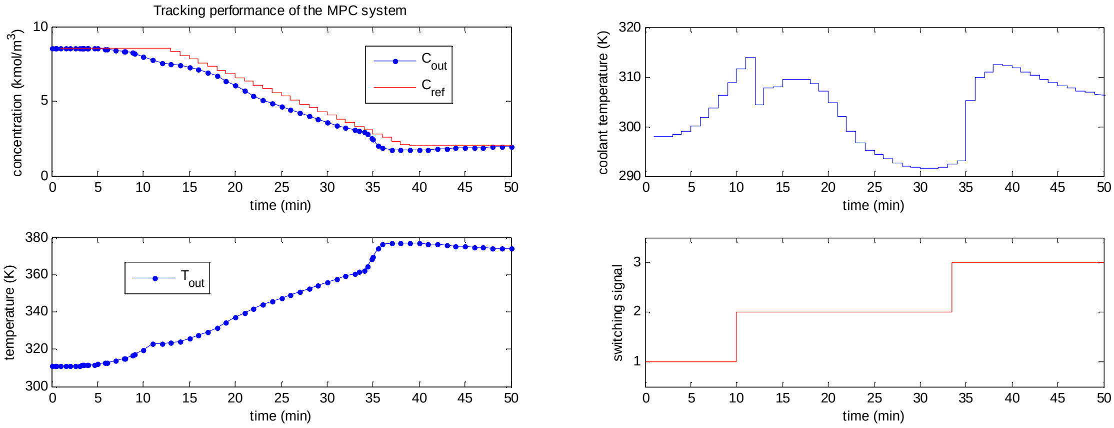

Figure 4. Concentration tracking, and coolant temperature adjustment as well as switching signal.

The residual concentration follows the desired trajectory relatively closely; although, some abrupt changes in the coolant temperature are observed at the time of switching. The IAE = 15.8

Full-State Composite Nonlinear Feedback Control

As regards to the chemical reactor, at least two possibilities exist for the CNF methodology. A CNF controller could be fabricated as a secondary controller within the previous cascade structure, or as a standalone full-state feedback controller. An immediate and interesting question emerges: Is it possible to perform the full-range control task using a full-state CNF controller with single tuning only? To answer this question the methods presented in /2–6/ are adopted for designing such control law. The purpose here is to demonstrate the ability of the full-state CNF controller to adjust the coolant temperature, without the need for gain-scheduling. This is the key feature of the CNF controller. The full-state CNF controller can be designed with respect to the linearized process dynamics described by the equations

$$\begin{cases} \dot{x} = Ax + Bu, \ x(0) = x_0 \\ y = C_y x \\ m = Ix \end{cases}, \tag{2}$$

where y is the controlled output, m is the measured state, u is the control input, x_0 is an initial condition, whereas I is an identity matrix of appropriate dimensions. It is further assumed that the pair (A, B) is stabilizable, and the triple (A, B, C_y) has no invariant zeros at the origin. A full-state CNF controller can be designed by the following three steps /2/.

Step 1. Design a linear feedback control law

$$u_L = -Kx + R_s r \tag{3}$$

such that (A–BK) is asymptotically stable. The scalar R_s is a reference tracking gain given by

$$R_s = -[C_y(A - BK)^{-1}B]^{-1}, \tag{4}$$

where the inner inverse exists under the given assumptions.

Step 2. Design a nonlinear feedback law

$$u_N = \rho(r, y)B^T Px, \tag{5}$$

where $\rho(r, y)$ is a nonpositive function, locally Lipschitz in y. General procedure for designing the matrix B^TP is presented; for example, in /3/. In this paper, the nonlinear function is chosen as

$$\rho(e) = -\beta\left|\exp(-\alpha|e|) - \exp(-\alpha|y(0) - r|)\right|, \ |e| = |r - y|, \tag{6}$$

where e is the control error, and $\alpha > 0$ and $\beta > 0$ are tuning parameters. /3/

Step 3. Form the full CNF control law by combining the linear and nonlinear feedback laws from the previous steps:

$$u = u_L + u_N = -Kx + R_s r + \rho(r, y)B^T Px. \tag{7}$$

The proof of the asymptotic stability of the closed-loop system under such control law can be found; for example, in /3/, respectively.

Since the reactor is nonlinear by nature, it is not possible to guarantee asymptotic stability in the entire state space by inspecting the linearized models only. Therefore, the stability is verified using the five design models used in the gain-scheduling design. A theoretical risk of instability in other points in state space exists as in the case for gain-scheduling. It should be noted that Lyapunov functions could be used to investigate the stability in the entire operation region of the nonlinear reactor. However, such investigation is not the concern of this study.

Following the Steps 1–3, results in the CNF controller. First, a linear feedback law is designed using pole-placement method such that both closed-loop eigenvalues are at –1 at the negative real axis. Such eigenvalues result in: $K = [-3.2118 \quad 13.002]$. Furthermore, the reference tracking gain $R_s = -3.8024$, which is designed based on the linear model of the final design node. Such R_s calibrates the DC-gain of the control system to unity, when the transition is complete. Next, a Lyapunov equation

$$(A - BK)^T P + P(A - BK) + Q = 0 \tag{8}$$

is solved for $Q = \mathrm{diag}\{5, 25\}$, which results in well-conditioned P given by

$$P = \begin{bmatrix} 8.9948 & -2.3966 \\ -2.3966 & 8.0004 \end{bmatrix} > 0,\ P = P^T \tag{9}$$

The nonlinear feedback gain is then: $K_n = B^T P = [-0.7190 \quad 2.4001]$. Finally, α = 3 and β = 1, which completes the CNF design. The block diagram of the full-state CNF topology is depicted in Figure 5, respectively.

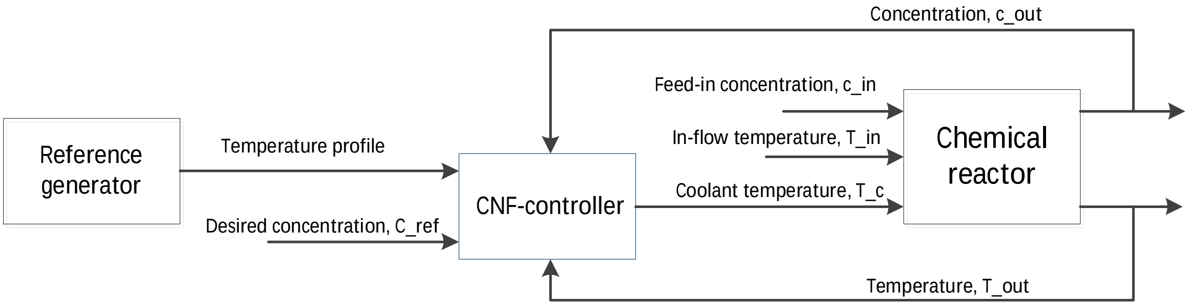

Figure 5. Full-state CNF topology for chemical reactor.

Based on the linearized models and the desired concentration reference, it is possible to calculate approximate values for the corresponding reactor temperatures at each design node. Furthermore, performing linear interpolation between the calculated target temperature values, an approximate temperature trajectory can be formed for the CNF controller. The resulting trajectory can be considered as an output of a reference generator. Such generator is depicted in Figure 5. In general, reference generators using auxiliary linear systems for the CNF methodology can be found in /5/, respectively. As regards to cascade control, the temperature reference is provided as the output of the primary controller.

The full-state CNF controller is refined with a smoothing filter between the target reference and the control signal. Such filter smooths out any possible abrupt change in the coolant temperature adjustment, when the concentration reference starts to ramp down from its initial value. Interested readers should refer to /9/ for the discussion of general feedforward filter design for the CNF controllers. The performance of the full-state CNF controller is depicted in Figure 6, respectively.

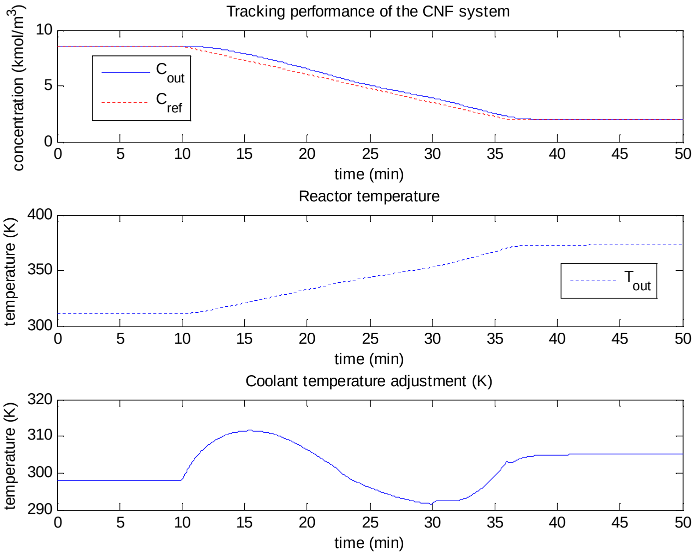

Figure 6. Performance of the CNF controller.

The concentration tracking is the best among the other two candidates. The CNF controller follows the desired reference with marginal error only, even though the generator shows imperfections in its trajectory; however, it does display the controller's capability to remove effects of modeling errors. Also, the coolant temperature change appropriately within the given bounds and the IAE = 10.7, which is substantially smaller compared to IAE's of other candidates.

4 CONCLUDING REMARKS

In this paper, a full-state CNF controller was designed for adjusting the coolant temperature of an exothermic chemical reactor process transitioning from low to high conversion rate. The reaction dynamics are highly nonlinear during the transition, and the reaction goes through a stable-unstable-stable chain when the residual concentration is ramped down from initial to final concentration. The rate of change of the coolant temperature was limited to 10 degrees/min, which effectively restricts the achievable performance of the overall tracking. The performance of the CNF controller using single tuning was compared to gain-scheduled PI-controller and P-controller within a cascade structure as well as to model predictive controller. It was shown by simulation that the CNF controller provided the best tracking performance as measured by the IAE.

5 ACKNOWLEDGMENT

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