An Introduction to Multiobjective Optimization

Karthik Sindhya

University of Jyvaskyla, Department of Mathematical Information Technology,

P. O. Box 35, FI-40014, University of Jyvaskyla, Finland

Tel: +358 405020418 Karthik.sindhya@jyu.fi , http://users.jyu.fi/~kasindhy/

KEY WORDS: MCDM, EMO, Pareto optimality

ABSTRACT

Multiobjective optimization in industry has rapidly grown in importance, as it provides the possibility for a designer or an engineer to consider the problem in hand as a whole. Solution to a multiobjective optimization problem involves several optimal solutions with different trade-offs. As a result upon optimization s(he) can understand the trade-offs between different solutions and subsequently choose the most preferred solution. In this paper, we provide a bird's eye view of the different methods available in the literature to solve multiobjective optimization problems. Specifically, in literature there exists at least two different research fields i.e. multiple criteria decision making and evolutionary multiobjective optimization. Here we present briefly an introduction to these two research fields. Thus this paper shall active as a catalyst for the growth of multiobjective optimization in diverse fields of engineering.

1 INTRODUCTION

Optimization problems in industry are often considered to be single objective e.g. maximization profit, purity etc. and often there exists only a single optimal solution to such problems. However optimization problems are rarely single objective, in fact multiple conflicting objectives e.g. maximize torque, minimize cost etc. in the design of motors exists. We commonly refer to such problems as multiobjective optimization problems (MOPs). Multiobjective optimization problems usually do not have a single optimal solution, instead multiple optimal solutions exists with different trade-offs. Since there are multiple optimal solutions, a decision maker (DM) who is an expert in the subject field of MOP is involved to choose her/his preferred optimal solution(s) among them. It must be noted that considering all possible and appropriate objectives within the problem formulation provides a comprehensive understanding of the problem and enables one to find her/his preferred solution among several optimal solutions. There exist at least two research fields which have concentrated on solving MOPs, i.e. multiple criteria decision making (MCDM) /1/ and evolutionary multiobjective optimization (EMO) /2/.

The field of MCDM dates back several decades /3/ and aims to mainly support the DM in finding her/his preferred optimal solution. Here often multiple conflicting objectives are converted in to a single objective problem considering the preferences of the DM and subsequently solved using an appropriate mathematical programming technique. In the field of EMO nature inspired algorithms e.g. genetic algorithms /4/, which are population based are used to obtain a set of solutions that approximate the set of optimal solutions using evolutionary principles. Here multiple conflicting objectives are considered simultaneously in the solution process /2/. In literature there exists several ways of classifying the methods used to solve MOPs. In this paper we use the four class based classification proposed in /1/, which is based on the role played by the DM in the solution process. The four classes include no-preference, a priori, a posteriori and interactive methods which are explained later in this paper. This paper presents a brief overview of the field of multiobjective optimization including MCDM and EMO methods and shall lay as a base for further study and research.

In this paper, we first present some basic concepts and terminologies in Section 2. Next, in Section 3 we present a classification of the methods available to solve MOP problems. Finally we conclude in Section 4.

2 PROBLEM DEFINITION AND IMPORTANT CONCEPTS

Let us consider MOP of the following form:

$$Minimize\ \{f_1(\boldsymbol{x}), f_2(\boldsymbol{x}), \dots, f_k(\boldsymbol{x})\}$$

$$\text{Subject to } \boldsymbol{x} \in S \subset \mathbb{R}^n,$$

with $k \geq 2$ objectives, $f_i : S \rightarrow \mathbb{R}$. If MOP involves objective function f_i to be maximized then we consider an equivalent objective function $-f_i$ that is minimized. Generally, a MOP has many optimal solutions called Pareto optimal solutions with different trade-offs. A decision vector $\boldsymbol{x}^* \epsilon\, S$ is Pareto optimal, if there does not exist any other $\boldsymbol{x} \epsilon\, S$, such that $f_i(\boldsymbol{x}) \leq f_i(\boldsymbol{x}^*)\ \forall\, i = 1, 2, \dots, k$ and $f_j(\boldsymbol{x}) < f_j(\boldsymbol{x}^*)$ for at least one index j. In simple words, a solution is Pareto optimal if no objective function value can be improved without impairing any other objective function values. An objective vector (k dimensional) is Pareto optimal if the corresponding decision vector is Pareto optimal. The set of Pareto optimal solutions in the decision space is called Pareto optimal set and their corresponding set of Pareto optimal solutions in the objective space is called Pareto optimal front.

It is also common to define two different objective vectors i.e. ideal and nadir points that provide ranges of objective function values in the Pareto optimal front. An ideal point is obtained by individually minimizing each of the objective functions subject to the constraints ($z_i^{ideal} = \min_{\boldsymbol{x}\, \epsilon\, S} f_i(\boldsymbol{x})\, , i = 1, 2, \dots, k$) and provides the lower bound of the objective function values of the Pareto optimal front. A nadir point provides the upper bound of the objective function values of the Pareto optimal front and is usually difficult to calculate. It is common to approximate the nadir point using a pay-off table /1/. Recently, methods are also proposed to find a reliable estimate of the nadir point /5/, but are difficult to implement. In addition to the ideal and nadir points, it is also common to define an additional objective vector called utopian objective vector, which is calculated as $z_i^{utopia} = z_i^{ideal} - e_i,\ e_i > 0, i = 1, \dots, k$. The importance of utopian point is limited to account for the numerical problems that arise when ideal and nadir points are close to each other.

As mentioned before, there exists several Pareto optimal solutions to a MOP and a DM is needed to choose her/his preferred Pareto optimal solution. There exists several ways for a DM to express her/his preference information /1/, /6/. A common way is to express the preference information in terms of desirable values of objective functions ($\overline{z_i}, i = 1,2, \dots, k$), often termed as a reference point. In the next section we present the classification of methods used to solve MOPs.

3 CLASSIFICATION OF MULTIOBJECTIVE OPTIMIZATION METHODS

Several ways of classification can be found in the literature. Here we present the classification followed in /1/, i.e. no preference methods, a posteriori methods, a priori methods, and interactive methods. This classification is based on the role played by the DM during the solution process. Also we briefly explain each of these methods with a few example methods that belong to them.

3.1 No-preference methods

When a DM is not in the position to provide any preference information or no DM is available, a MOP is solved to obtain a Pareto optimal solution that is closer to the ideal point. One of the most commonly used methods that belong to no-preference method is the method of global criterion /7/. In the method of global criterion, the MOP is converted in to the following single objective optimization problem:

$$Minimize\ (\sum_{i=1}^{k} \left| f_i(\boldsymbol{x}) - z_i^{ideal} \right|^p)^{1/p}$$

$$\text{subject to } \boldsymbol{x} \in S.$$

In the above single objective problem, any $p \leq \infty$ can be used. Different Pareto optimal solutions can be obtained by changing the value of p, as the closeness measure to the ideal point is varied /1/.

3.2 A posteriori methods

As the name suggests in a posteriori methods, the DM is involved only after the MOP is solved. Here solving a MOP means finding a representative set of Pareto optimal solutions or the Pareto optimal front. The DM investigates the Pareto optimal front presented to her/him and then chooses a solution from them as her/his preferred Pareto optimal solution. Different methods are employed to obtain the representative Pareto optimal front, i.e. weighted sum method /8/, epsilon constraint method /9/, EMO algorithms /2/ etc. Among these, EMO algorithms are commonly used.

Evolutionary multiobjective algorithms are based on nature inspired algorithms and involve a population of solutions that are evolved over several iterations or generations to finally obtain a representative Pareto optimal front. There are at least two main goals in EMO, to obtain a set of solutions as close as possible to the Pareto optimal front and to find a set of diverse solutions that represent the entire Pareto optimal front. In the above goal closeness is relevant as EMO algorithms have no theoretical convergence proof to optimal solutions. The main convergence criterion that is employed is maximum number of function evaluations. In EMO algorithms, a representative solution is called an individual, a set of individuals is called a population and an iteration of an EMO algorithm is called a generation. Typically in an EMO algorithm, a set of new individuals are randomly created. Subsequently, new solutions are created using crossover and mutation operators /2/ commonly called reproduction operators. Next a selection operator is applied to prefer good solutions over bad solutions. Good solutions are usually those that have better function values as compared to other solutions and/or can help in maintaining the diversity of solutions (second goal of EMO). This combination of crossover, mutation and selection operation forms one generation of an EMO algorithm. Several such generations are carried out until a pre-fixed termination criterion is met. Finally the resulting population (which constitutes solutions that are equally good) is declared to be the representative Pareto optimal front.

In EMO literature there are several EMO algorithms and are often classified in to three main groups, i.e. aggregation based, dominance based and performance indicator based algorithms. Aggregation based algorithms decompose MOP into a number of single objective sub-problems and subsequently solve them simultaneously. One of the commonly used aggregation based EMO algorithm is MOEA/D /10/. Dominance based algorithms are the most common type of EMO algorithms in the literature and are based on the Pareto domance based evaluation /2/ of individuals. A commonly used dominance based EMO algorithm is NSGA-II /11/. Finally, indicator based algorithms /12/ use an indicator function, e.g. hypervolume (a measure of the area dominated by the Pareto optimal front) to gauge the quality of the population in an EMO algorithm.

Over the past few decades EMO algorithms have been increasingly used in practice due to advantages such as, a set of representative Pareto optimal solutions can be obtained in a single run, multiple local, discrete, and nonconvex

Pareto optimal fronts and different types of variables, objective functions, and constraints can be easily handled. However they are also equally criticized for their lack of convergence proof, being computationally very expensive and the need to set a number of algorithm parameters such population size etc.

3.3 A Priori methods

In a priori methods, a DM provides her/his preference information beforehand. This preference information is considered to usually formulate a single objective optimization problem, which is subsequently solved to obtain preferred Pareto optimal solutions to the DM. Although this method looks attractive, in practice it is often difficult for a DM to know a priori what he wishes to achieve. Thus the preference information provided by the DM can be too optimistic or pessimistic. Lexicographic ordering /13/ and goal programming /14/ are commonly used a priori methods.

In lexicographic ordering, the DM ranks the objective functions in the order of importance. Firstly the most important objective function is optimized subject to constraints. Next, if the solution to this solved problem is not unique, then the second objective function is optimized subject to the constraint that the objective function value of the most important objective function is no less than the optimal value. This procedure continues until we obtain a unique solution. In goal programming the DM provides a goal in terms of the value of objective function s(he) wishes to achieve. Subsequently a solution is obtained by minimizing the deviation between the feasible objective function value and the specified goals.

3.4 Interactive methods

Interactive methods are the most DM intensive. Here the DM articulates her/his preference information iteratively and thus directs the MOP solution process progressively. A typical interactive method starts with showing the DM a starting Pareto optimal solution with the ideal and nadir points. Next, the DM provides her/his preference information, which is subsequently used by the interactive method to generate one or more Pareto optimal solutions. The DM again investigates the solutions provided to her/him and chooses one or more solutions that s(he) likes and provides new preference information. The procedure of expressing preference information and subsequently finding new solutions corresponding to the preference information continues until the DM has found her/his preferred Pareto optimal solutions and does not wish to continue.

Literature has a plethora of interactive methods proposed in the literature, such as STEP method /15/, NIMBUS method /16/ etc. NIMBUS method proposed by Miettinen and Mäkelä /16/ is perhaps one of the well know interactive methods available in the literature. It closely follows the steps involved in a typical interactive method mentioned above. When the DM is expressing her/his preference information, s(he) classifies the objectives in to one of five class indicating how the current solution can be improved, i.e. the value of objective function a) have to be improved from the current value, b) have to be improved up to a specified value by the DM, c) is acceptable and must stay the same, d) have to impaired up to specified value by the DM and e) can vary freely temporarily. This information is used by the NIMBUS method to formulate a single objective function and a new Pareto optimal solution is obtained. Further details about the method can be obtained in /16/. Usually in engineering design the objective and constraint functions are computationally expensive to evaluate, this limits the applicability of interactive methods in such cases. As the DM has to wait a long period of time to investigate even one solution.

4 Conclusions

In this paper, we present a brief overview of the methods available in the literature. Classifications of the methods are presented and a brief explanation of at least one method is presented. This could provide an overview to researchers and practitioners about the field of multiobjective optimization as a whole and thereby promote its utilization among the industries. Typically only multiobjective optimization problems with only a couple of objectives are considered in industries, in such a case the use of EMO algorithms are common in the literature. This is mainly due to the ease of visualization of the Pareto optimal front. However, if several objectives are considered it is advisable to use interactive methods, as the DM can iteratively present his preferences and learn about the problem before converging to her/his preferred Pareto optimal solution.

References

1. Miettinen, K.: Nonlinear Multiobjective Optimization, Kluwer, Boston, 1999.
2. Deb, K.: Multi-objective Optimization using Evolutionary Algorithms, Wiley, Chichester, 2001.
3. Köksalan, M., Wallenius, J., Zionts, S.: Multiple Criteria Decision Making, from Early History to the 21st Century, World Scientific, 2011.
4. Goldberg, D.E.: Genetic Algorithms for Search, Optimization, and Machine Learning, Addison-Wesley, Reading. 1989.
5. Deb, K., Miettinen, K., Chaudhuri, S.: Toward an estimation of nadir objective vector using a hybrid of evolutionary and local search approaches. IEEE Transactions on Evolutionary Computation, 14(2010) 6, 821–841.
6. Luque, M., Miettinen, K., Eskelinen, P., Ruiz, F.: Incorporating preference information in interactive reference point methods for multiobjective optimization. Omega, 37(2009) 2, 450-462.
7. Yu, P. L.: A class of solutions for group decision problems. Management Science, 19(1973) 8, 936–946.
8. Gass, S., Saaty, T.: The computational algorithm for the parametric objective function. Naval Research Logistics Quarterly, 2(1955) 1-2, 39–45.
9. Haimes, Y. Y., Lasdon, L. S., Wismer, D. A.: On a bicriterion formulation of the problems of integrated system identification and system optimization. IEEE Transactions on Systems, Man and Cybernetics, 1(1971) 3, 296–297.
10. Zhang, Q., Li, H.: MOEA/D: A multiobjective evolutionary algorithm based on decomposition. IEEE Transactions on Evolutionary Computation, 11(2007) 6, 712-731.
11. Deb, K., Pratap, A., Agarwal, S., Meyarivan, T.: A fast and elitist multiobjective genetic algorithm: NSGA-II. IEEE Transactions on Evolutionary Computation, 6(2002) 2, 182-197.
12. Beume, N., Naujoks, B., and Emmerich, M.: SMS-EMOA: multiobjective selection based on dominated hypervolume. European Journal of Operational Research 180(2007) 3, 1653-1669.
13. Fishburn, P. C.: Lexicographic orders, utilities and decision rules: A survey. Management Science, 20(1974) 11, 1442–1471.
14. Charnes, A., Cooper, W. W.: Goal programming and multiple objective optimizations: Part 1. European Journal of Operational Research, 1(1977) 1, 39–54.
15. Benayoun, R., De Montgolfier, J., Tergny, T., Laritchev, O.: Linear programming with multiple objective functions: Step method (STEM). Mathematical programming, 1(1971) 1, 366–375.
16. Miettinen, K., Mäkelä, M.: Interactive bundle-based method for nondifferentiable multiobjective optimization: NIMBUS. Optimization, 34(1995), 231–246.