Experiment Designs for Control-Oriented MIMO Identification

Kurt-Erik Häggblom
Åbo Akademi University, Faculty of Science and Engineering, FI-20500, Åbo (Turku)
Tel: +358 2 215 4447, E-mail: khaggblo@abo.fi

Ramkrishna Ghosh
Åbo Akademi University, Faculty of Science and Engineering, FI-20500, Åbo (Turku)
Tel: +358 2 215 3525, E-mail: rghosh@abo.fi

KEY WORDS system identification, input design, multivariable systems, integral controllability

ABSTRACT

In this paper, guidelines for designing inputs for control-oriented identification of multivariable systems are given. The information needed for design of inputs such as double rectangular pulses, pseudo-random sequences (PRBS), and multi-sinusoidal signals can be obtained from a simple step test. Such a step test also gives accurate information about the directionality properties of the system. It is shown how this information can be used to design input signals as well as directional inputs that explicitly excite the various gain directions of the system. By such excitations, small gains and fast dynamics can be better identified than by standard, non-directional, methods. If the identified model is to be used for control design, it is important that such properties are included in the model. The design techniques are illustrated by simulations on an ill-conditioned, 3-input, 3-output system. In addition to output noise, low-frequency input disturbances are added to make the identification task realistic with regard to real-world applications.

1 INTRODUCTION

A successful system identification requires data that are truly representative of the system to be identified. To obtain such data, the experiment design for the identification is of utmost importance. In this respect, multiple-input multiple-output (MIMO) systems are much more challenging than single-input single-output (SISO) systems.

One approach to MIMO system identification is to perturb on input after another and to identify each input-output dynamics separately as a set of SISO models /1/. All outputs can also be considered simultaneously, resulting in a SIMO (single-input, multiple-output) approach. However, it is clear that parameters common to more than one SISO or SIMO subsystem will then be identified as being different, which generally makes the order of the overall model higher than the order of the system.

In order to minimize the experiment time, it is beneficial to perturb all inputs simultaneously. This puts high demands on the experiment design. The standard approach is to use uncorrelated signals, e.g., pseudo-random binary sequences (PRBS), for the various inputs. The identification task could then be handled as a set of MISO (multiple-input, single-output) problems by treating each output separately. However, this approach has similar drawbacks as the SIMO approach.

If the identified model is to be used for simulation, the above approaches are probably adequate. However, for prediction and control applications, it is essential that all outputs are treated simultaneously /2/. The reason is that correlations between the outputs (i.e., “directionality”) are not accounted for by a multi-MISO approach /3/. Thus, a MIMO system should be identified as a full MIMO system with all inputs and all outputs handled simultaneously. The general view is that the inputs should then be uncorrelated to ensure identifiability.

In this paper, guidelines are given for design of identification experiments aimed at fulfilling the above mentioned requirements for MIMO system identification. Special emphasis is put on control-oriented aspects. If a model is used for control design, integral controllability requires that the directionality properties or the model are close to those of the real system.

The design techniques are illustrated by realistic simulations of an ill-conditioned 3×3 system. Previously, a similar study of a 2×2 system with real data from a pilot-scale distillation column was made /4/. A simulated 4×4 system has also been studied /5/.

2 CONTROL-ORIENTED INPUT DESIGNS

2.1 Integral Controllability

A multivariable controller with integral action, such as a model-predictive controller (MPC), can stabilize two systems having gain matrices K and $\hat{K}$, respectively, if and only if /6, 3/

$$\mathrm{Re}[\lambda_i(K\hat{K}^{-1})] > 0\,,\ \forall i\,, \tag{1}$$

where $\lambda_i(\cdot)$ is the ith eigenvalue of $(\cdot)$. If the system to be controlled has the gain matrix K and the model used for controller design has the gain matrix $\hat{K}$, (1) must obviously hold. A necessary condition for this is that the determinants of K and $\hat{K}$ have the same signs.

An ill-conditioned system is a MIMO system whose gain matrix has a "high" condition number /7/. Such a matrix is nearly singular, which means that small errors in $\hat{K}$ may give large errors in $\hat{K}^{-1}$. This may easily cause (1) to be violated. However, if the directions of corresponding column vectors in K and $\hat{K}$ are reasonably close to each other, even large errors in the magnitudes of the column vectors can be tolerated without (1) being violated. Thus, it is desirable that possible errors in $\hat{K}$ are closely aligned with the column vectors of K.

In system identification, the distribution of errors in $\hat{K}$ primarily depends on the quality of the data. Besides reliable measurements, the only way of ensuring good data is the experiment design, i.e., the applied inputs in the identification experiments. As a minimum, a control-oriented experiment design aims at producing data that enables (1) to be satisfied.

It has been recognized that proper excitation of the so-called gain directions of the system is a good way of generating data for a control-oriented identification /3/. Although the design is based on steady-state considerations only, there are many ways of implementing such a procedure /4/. The fact that the slow and fast dynamics of a system tend to be well separated among the gain directions makes the method very effective for identification of dynamics, too /8/.

2.2 Directional Input Design

Consider a system with an input vector u, an output vector y, and a non-singular steady-state gain matrix K of size $n\times n$. A singular value decomposition (SVD) of K yields

$$\bar{y} = K\bar{u} = W\Sigma V^{\mathrm{T}}\bar{u}\,, \tag{2}$$

where $\bar{u}$ and $\bar{y}$ denote steady-state values. V and W are orthogonal matrices and Σ is a diagonal matrix of singular values, σ_i, $i=1,\ldots,n$, $\sigma_1 \ge \sigma_2 \ge \ldots \ge \sigma_n > 0$. The orthogonality means that $V^{\mathrm{T}}V = I$ and $W^{\mathrm{T}}W = I$.

A new signal is defined by

$$\xi = \Sigma V^{\mathrm{T}} u\,. \tag{3}$$

The steady-state output is then given by

$$\bar{y} = W\bar{\xi}\,. \tag{4}$$

Because W is given by the SVD, $\bar{\xi}_i$ (i.e., the ith component of $\bar{\xi}$) will excite only the output direction associated with the singular value σ_i resulting in an output with the steady-state magnitude $\|\bar{y}\|_2 = |\bar{\xi}_i|$.

Of course, ξ cannot be applied directly as an input, but it can be realized (approximatively) by the input

$$u = \hat{V}\hat{\Sigma}^{-1}\xi\,, \tag{5}$$

where $\hat{V}$ and $\hat{\Sigma}$ are estimates of V and Σ, respectively, determined from an estimate $\hat{K}$ of K. If $V^{\mathrm{T}}\hat{V} \approx I$, substitution of (4) into (2) yields

$$\bar{y} \approx W\Sigma\hat{\Sigma}^{-1}\bar{\xi} = \sum_{i=1}^{n} w_i \sigma_i \hat{\sigma}_i^{-1} \bar{\xi}_i \,, \tag{6}$$

where w_i is the ith column of W. Although the directional design is based on steady-state considerations, (5) shows how the input u should be varied in time when the design signal ξ varies in time.

There are numerous design options for ξ. It is possible to excite one gain direction (and the associated dynamics) at a time by perturbing one component ξ_i at a time. Equation (6) shows that an accurate estimate of σ_i is then not that important as the estimate $\hat{\sigma}_i$ only affects the magnitude of the steady-state output vector $\bar{y}$. It is also possible to excite all gain directions simultaneously by perturbing all components of ξ simultaneously. The perturbations ξ_i should then be *uncorrelated* with each other to make the various gain directions uncorrelated and thus identifiable. *Note that this is different from applying uncorrelated inputs u_i as perturbations.*

Independently of the above choice, ξ_i can be any type of excitation signal normally used as input in identification. It can, e.g., be a (series of) step change(s), a double rectangular pulse (DRP), a pseudo-random binary sequence (PRBS), or a multi-sinusoidal signal (MSS). The signals for the various directions can be designed with dynamics in mind to excite different frequency ranges. In principle, it is even possible to use different types of signals for the various gain directions.

2.3 Test Signal Design

A proper design of test signal requires some knowledge of the frequency range of interest. This information may be obtained from an initial step test. Such a test will also give an estimate of the gain matrix needed for a directional input design.

In the following, design principles for a DRP, a PRBS, and a MSS are given. They are mainly based on specifications (estimates) of the smallest time constant of interest, T_{L}, and the largest time constant of interest, T_{H}. The signal amplitude a is left as a tuning parameter for desired output magnitudes.

2.3.1 Double rectangular pulse

A DRP is composed of a rectangular pulse with height a and duration T_{sw} directly followed by a similar pulse with height $-a$. The DRP excites almost all frequencies of a system, with the maximum excitation at a higher frequency than the low frequencies mainly excited by a step signal.

There seems to be no readily available design rules for the DRP in the open literature. The signal properties could be analysed via the Fourier transform to obtain some guidelines, but here we suggest the simple choice

$$T_{\mathrm{sw}} \approx T_{\mathrm{M}} \,, \tag{7}$$

where T_{M} denotes the main time constant of interest. This approximately corresponds to a sine wave with the period T_{M}.

After the DRP, the signal, denoted $u_{\mathrm{drp}}(t)$, remains at 0 for the remaining experiment time. For directional inputs, one such signal is used to excite one gain direction, suitably time-shifted versions of the signal are used for other gain directions. Thus, one direction at a time is excited. Let the time delay chosen for direction " i " be θ_i, and the amplitude chosen to properly excite this direction be a_i. Component ξ_i of the design signal for directional inputs then becomes

$$\xi_i = a_i u_{\mathrm{drp}}(t-\theta_i) \,,\ i = 1,\ldots,n \,. \tag{8}$$

The full design signal ξ is applied to the system by (5).

2.3.2 Pseudo-random binary sequence

A PRBS is a deterministic binary signal with a sequence length N. Repeated sequences give a periodic signal. The signal switches between the levels a and $-a$ with a minimum switching time T_{sw} such that the time between switches is some multiple of T_{sw}. By design, the sample statistics of the signal accurately mimic those of white noise within the sequence length.

There are many ways of implementing a PRBS. The most common version is a maximum-length PRBS, for which the period length satisfies

$$N = 2^{n_{\mathrm{r}}} - 1\,, \tag{9}$$

where n_{r} is a positive integer, the so-called register length. With N specified, the switching times can be calculated by a simple formula /2/, but here, MATLAB's System Identification Toolbox /9/ will be used.

The design principles are as follows. If the highest angular frequency of interest is $\omega_{\max}$ with the period length T_{L}, T_{sw} should be selected as /10/

$$T_{\mathrm{sw}} \approx 2.5\omega_{\max}^{-1} \approx 0.4T_{\mathrm{L}}\,. \tag{10}$$

The frequency $\omega_{\max}$ can often be taken as the bandwidth of the system, or of the closed-loop system to be designed.

The total period length NT_{sw} determines the low-frequency excitation. It has been suggested to select NT_{sw} as the 99 % settling time of a step response /1/. If the largest time constant of interest is T_{H}, the recommendation is

$$N \approx \beta T_{\mathrm{H}} / T_{\mathrm{sw}}\,. \tag{11}$$

where β is chosen according to the desired settling time. For a 95 % settling time, $\beta = 3$; for 99 %, $\beta = 4.6$. Note however, that N has to be selected to satisfy (9). Compromises might also be needed to keep the experiment length NT_{sw}, or some multiple of it, sufficiently short.

The minimum switching time should not be taken as the sampling time. In fact, it is recommended /2/ that the sampling time T_{s} be selected as

$$T_{\mathrm{s}} \approx 0.25T_{\mathrm{sw}}\,. \tag{12}$$

Together with (10), this corresponds to 10 samples per T_{L}.

For directional inputs, the same PRBS, suitably time-shifted to make them uncorrelated, can be used for all gain directions unless there is a reason to design them for different dynamics. If the PRBS signal is denoted $u_{\mathrm{prbs}}(t)$,

$$\xi_i = a_i u_{\mathrm{prbs}}(t - \theta_i)\,,\ i = 1,\ldots,n\,. \tag{13}$$

Usually, the time shifts are selected as $\theta_i = NT_{\mathrm{sw}}(i-1)/n$, $i = 1,\ldots,n$. This is the choice used in the Identification Toolbox. However, for $n \geq 3$, other time shifts might yield smaller correlations between the signals.

2.3.3 Multi-sinusoidal signal

A MSS has the form

$$u_{\mathrm{mss}}(t) = a\sum_{k=1}^{n_{\mathrm{s}}} \cos(\omega_k t + \phi_k)\,, \tag{14}$$

where n_{s} is the number of sinusoids, all (in this case) with the same amplitude a. The individual sinusoids have the frequency ω_k and phase shift ϕ_k, $k = 1,\ldots,n_{\mathrm{s}}$. A so-called Schroeder multi-sine uses the phase shifts /10/

$$\phi_k = -k(k-1)\pi / n_{\mathrm{s}}\,. \tag{15}$$

These phase shifts prevent the amplitudes of the sinusoids to add up excessively in the summation.

The remaining user choices are n_{s} and ω_k, $k = 1,\ldots,n_{\mathrm{s}}$. Assuming $\omega_1 < \omega_2 < \ldots < \omega_{n_{\mathrm{s}}}$, ω_1 is the lowest and

ω_{n_s} is the highest excitation frequency. The obvious choices are

$$\omega_1 \approx \frac{2\pi}{\beta T_\mathrm{H}} \ , \ \ \omega_{n_\mathrm{s}} \approx \frac{2\pi}{T_\mathrm{L}} \ . \tag{16}$$

Usually, equally spaced frequencies are desired. This requires

$$n_\mathrm{s} \approx \beta T_\mathrm{H} / T_\mathrm{L} \ . \tag{17}$$

The frequencies are then calculated by

$$\omega_k = \frac{2\pi k}{n_\mathrm{s} T_\mathrm{L}} \ , \quad k = 1, \ldots, n_\mathrm{s} \ . \tag{18}$$

This way of formulating the calculation ensures that the highest frequency is “exact”, the lowest frequency may be approximate depending on the approximation in (17).

For directional inputs,

$$\xi_i = a_i u_\mathrm{mss}(t - \theta_i) \ , \ i = 1, \ldots, n \ . \tag{19}$$

The time shifts θ_i to make ξ_i , $i = 1, \ldots, n$, uncorrelated, have to be found by analysing (13).

3 EXPERIMENTS

The presented input design methods are illustrated by realistic simulations of a 3×3 system.

3.1 Experimental Setup

The system used for this case study has the transfer function

$$G(s) = \begin{bmatrix} \dfrac{6\,\mathrm{e}^{-5s}}{22\,s+1} & \dfrac{20\,\mathrm{e}^{-5s}}{337\,s+1} & \dfrac{-1\mathrm{e}^{-5s}}{10s+1} \\ \dfrac{8\,\mathrm{e}^{-5s}}{50\,s+1} & \dfrac{77\,\mathrm{e}^{-3s}}{28s+1} & \dfrac{-5\,\mathrm{e}^{-5s}}{10\,s+1} \\ \dfrac{9\,\mathrm{e}^{-5s}}{50\,s+1} & \dfrac{-37\,\mathrm{e}^{-5s}}{166\,s+1} & \dfrac{-103\mathrm{e}^{-4s}}{23\,s+1} \end{bmatrix} . \tag{20}$$

The system was originally presented by Vasnani /11/, but an input and an output have been rescaled to make the system ill-conditioned. The gains and time constants (dimensionless numbers are used) have been rounded to the nearest integer. The condition number of the system is now 30. A SVD of the gain matrix gives

$$\Sigma = \begin{bmatrix} 114 & 0 & 0 \\ 0 & 74.8 & 0 \\ 0 & 0 & 3.78 \end{bmatrix}, \ V = \begin{bmatrix} -0.047 & 0.158 & 0.986 \\ 0.544 & 0.832 & -0.107 \\ 0.838 & -0.532 & 0.125 \end{bmatrix} . \tag{21}$$

Note that the outputs should be scaled to make equal numerical changes equally significant. Only then can norm-based measures of the output vector have relevance. The same applies to the inputs, if they are characterized by their norm. It is assumed that the signals now fulfil these requirements.

If the system is free of unmodelled disturbances, it is trivial to identify an exact model of the system with any reasonable input design, even simple step tests. White output noise will only marginally affect the estimated parameter values if the signal-to-noise ratio is reasonable good. For a more realistic simulation, it is necessary to add coloured noise, low-frequency disturbances, or nonlinearities. In this case, low- frequency, Schroeder multi-sines ($\omega_k = 2\pi k / 2550$, $k = 1,2,3;$ $a_1 = 0.05$, $a_2 = a_3 = 0.005$) were added to the inputs. White noise with (approximate) covariance 0.4 was added to all outputs.

Amplitudes of individual input signals (u_i or ξ_i , $i = 1,2,3$) were adjusted to render output vectors of approximately equal 2-norm and jointly scaled further to maximize outputs in the range $(-20, 20)$. In practice, this can hardly be done very precisely, but here the objective was to make comparisons of different input designs fair. For the same reason, the true model parameters were used as initial estimates in all identifications (but auto-selected values seemed to work equally well). Otherwise, the system was assumed to be unknown. MATLAB's System Identification Toolbox /9/ was used for all identifications.

3.2 Initial Step Test

A simple step test was performed to gain basic information about the system. The inputs were changed one at a time well separated to allow the system to reach a near steady state between changes. The input having the fastest dynamics was changed first, slowest dynamics last, to minimize the experiment duration. The step test is shown in Figure 1.

The identified model is shown in Table 1. The estimated gains $\hat{K}_{12}$, $\hat{K}_{13}$, and $\hat{K}_{23}$ differ considerably from the true ones and $\hat{K}_{13}$ even has the wrong sign. The condition number is 46 which is 50 % higher than the true one. A SVD of the gain matrix gives

$$\hat{\Sigma} = \begin{bmatrix} 108 & 0 & 0 \\ 0 & 76.3 & 0 \\ 0 & 0 & 2.33 \end{bmatrix}, \ \hat{V} = \begin{bmatrix} -0.036 & 0.156 & 0.987 \\ 0.574 & 0.812 & -1.108 \\ 0.818 & -0.563 & 0.119 \end{bmatrix}. \tag{22}$$

In spite of all this, the gain directions v_i, $i = 1,2,3$, are very close to the true ones. This $\hat{\Sigma}$ and $\hat{V}$ are used in the design of directional input.

3.3 Double Rectangular Pulse Experiment

The time constants of the estimated model vary between 0 and 638, but the extreme values, which differ considerably from the true ones, are associated with the very erroneous gains. However, in a real situation, this will not be known.

If the main time constant of interest is chosen as $T_M \approx 200$, the switching time for a DRP becomes $T_{\mathrm{sw}} = 200$. Figure 2 shows a directional DRP experiment, where the inputs are calculated by (8) and (5). The identified model is shown in Table 1.

3.4 PRBS Experiments

The DRP experiment might be expected to give a better model than the step test. However, the directionality data and the time constants from the step test model will be used to allow a fair comparison between the experiment designs.

If the almost zero time constants are not taken into account, the step test model suggests $T_{\mathrm{L}} \approx 25$, which yields $T_{\mathrm{sw}} = 10$ by (10). The sampling time according to (12) is rounded to $T_{\mathrm{s}} = 2$. Choosing $T_{\mathrm{H}} \approx 500$ and a 99% settling time ($\beta = 4.6$), (11) with the constraint (9), gives $N = 255$.

The command `idinput` in the System Identification Toolbox was used to generate a PRBS with $N = 255$. The command can also generate time-shifted PRBS signals, but it was found that adding three such signals produced

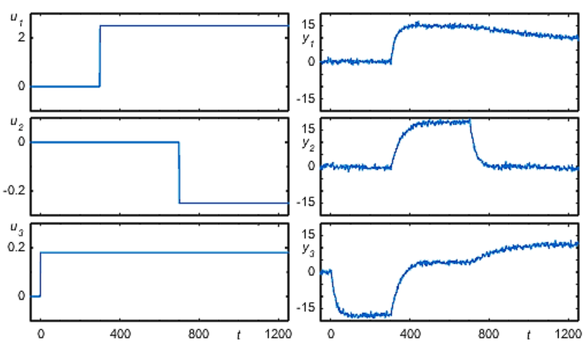

Figure 1. Initial step test.

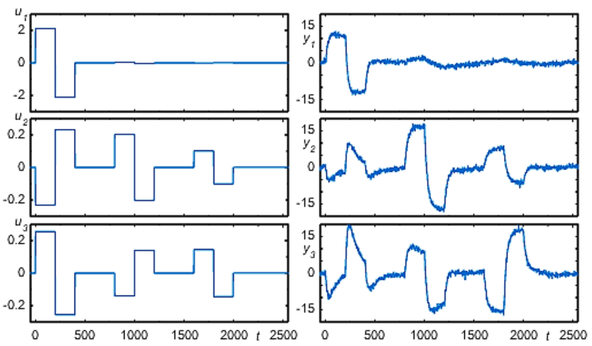

Figure 2. Directional DRP experiment.

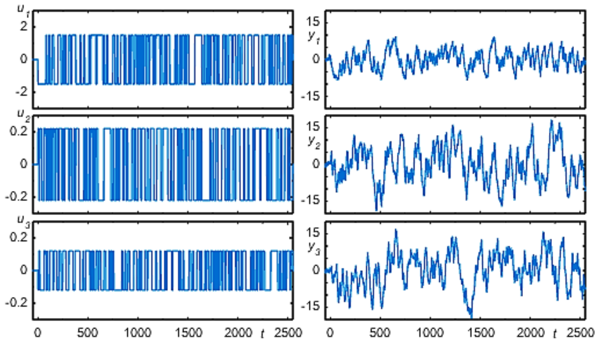

Figure 3. Uncorrelated PRBS experiment.

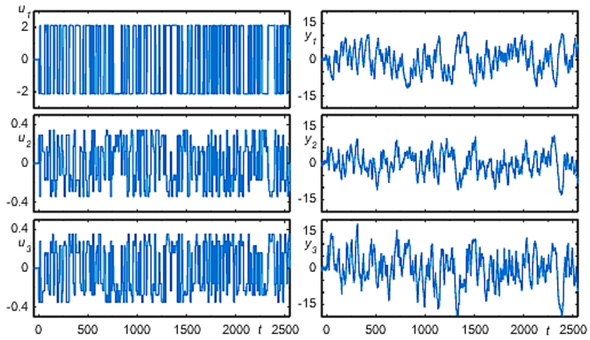

Figure 4. Directional PRBS experiment.

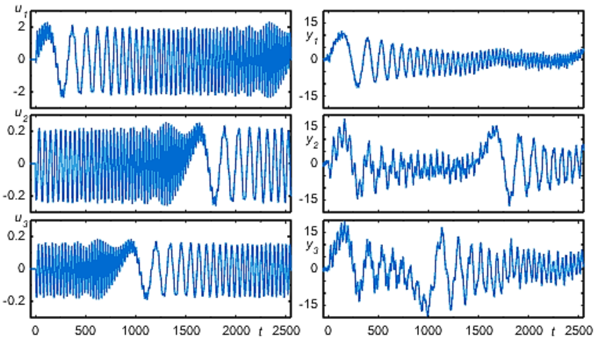

Figure 5. Uncorrelated multi-sine experiment.

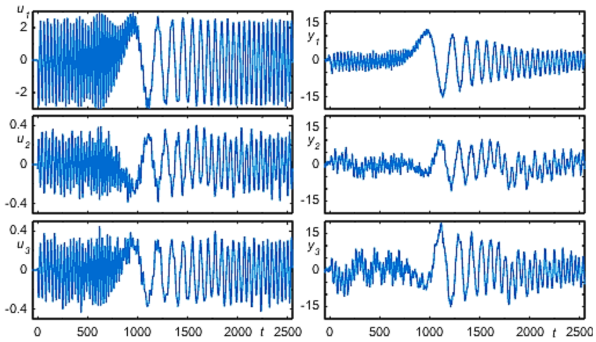

Figure 6. Directional multi-sine experiment.

a non-symmetrical signal with no values reaching $-3a$ or $3a$. By using other time shifts ($\theta_2 = 91$ and $\theta_3 = 179$), uncorrelated sequences were obtained that gave a symmetrical signal reaching the theoretical limits when added.

Figure 3 shows an experiment with uncorrelated PRBS inputs. Figure 4 shows an experiment with directional PRBS inputs calculated by (13) and (5). The obtained models are given in Table 1.

3.5 Multi-Sine Experiments

For the multi-sine design, $T_\mathrm{L} = 25$, $T_\mathrm{H} = 500$, and $\beta = 4.6$ are used in accordance with the PRBS design. Equation (17) gives $n_\mathrm{s} = 92$, which yields the period length $n_\mathrm{s} T_\mathrm{L} = 2300$. This is slightly shorter than the period length $NT_\mathrm{sw} = 2550$ used for the PRBS design. To get the same period length for both designs, n_s is increased to 102. The frequencies are then calculated by (18) with $n_\mathrm{s} T_\mathrm{L} = 2550$. For uncorrelated inputs, the time shifts $\theta_2 = 800$ and $\theta_3 = 1570$ were used.

Figure 5 shows an experiment with uncorrelated Schroeder multi-sine inputs calculated by (14), (15), and (18). Figure 6 shows an experiment with directional inputs calculated by (19) and (5).

4 RESULTS

4.1 Transfer Functions

The transfer function matrices of the estimated models are presented in Table 1.

It is clear that the transfer functions G_{12}, G_{13}, and G_{23} are very troublesome. Only the directional DRP and PRBS experiments gave reasonable estimates of these transfer functions. For the other transfer functions, there is not much difference between uncorrelated and directional excitations. The result of the multi-sinusoidal excitations is not that good.

Table 1. Estimated transfer functions.

Exp.	Uncorrelated	Directional
Step DRP	$\frac{5.8e^{-4s}}{22s+1}$ $\frac{1}{638s+1}$ $0.84e^{-79s}$	$\frac{6.3e^{-5s}}{24s+1}$ $\frac{13e^{-16s}}{148s+1}$ $\frac{-1.3}{9s+1}$
	$\frac{7.5e^{-6s}}{48s+1}$ $\frac{74e^{-3s}}{28s+1}$ $-2.7e^{-17s}$	$\frac{9.0e^{-1s}}{60s+1}$ $\frac{79e^{-2s}}{31s+1}$ $\frac{-6.1e^{-4s}}{9s+1}$
	$\frac{8.5e^{-6s}}{44s+1}$ $\frac{-30e^{-13s}}{137s+1}$ $\frac{-98e^{-4s}}{22s+1}$	$\frac{9.4e^{-4s}}{51s+1}$ $\frac{-58}{352s+1}$ $\frac{-104e^{-3s}}{23s+1}$
PRBS	$\frac{5.6e^{-5s}}{21s+1}$ $\frac{14e^{-7s}}{365s+1}$ $\frac{-0.12e^{-7s}}{10^4s+1}$	$\frac{5.9e^{-5s}}{22s+1}$ $\frac{12e^{-30s}}{362s+1}$ $\frac{-1.7e^{-9s}}{10s+1}$
	$\frac{7.5e^{-5s}}{47s+1}$ $\frac{74e^{-3s}}{27s+1}$ $\frac{-3.3e^{-7s}}{7s+1}$	$\frac{7.3e^{-5s}}{46s+1}$ $\frac{68e^{-3s}}{25s+1}$ $\frac{-8.2e^{-4s}}{22s+1}$
	$\frac{8.0e^{-5s}}{43s+1}$ $\frac{-54e^{-5s}}{186s+1}$ $\frac{-99e^{-4s}}{22s+1}$	$\frac{8.0e^{-5s}}{43s+1}$ $\frac{-66e^{-5s}}{155s+1}$ $\frac{-108e^{-4s}}{23s+1}$
MSS	$\frac{6.2e^{-5s}}{23s+1}$ $\frac{44e^{-5s}}{598s+1}$ $\frac{-0.64e^{-9s}}{0.5s+1}$	$\frac{6.0e^{-5s}}{21s+1}$ $\frac{81e^{-5s}}{598s+1}$ $\frac{-2.5e^{-9s}}{3473s+1}$
	$\frac{8.6e^{-5s}}{54s+1}$ $\frac{77e^{-3s}}{28s+1}$ $\frac{-4.5e^{-6s}}{8s+1}$	$\frac{9.2e^{-5s}}{55s+1}$ $\frac{83e^{-3s}}{29s+1}$ $\frac{-3.3e^{-6s}}{6s+1}$
	$\frac{9.7e^{-5s}}{55s+1}$ $\frac{-20e^{-7s}}{74s+1}$ $\frac{-105e^{-4s}}{24s+1}$	$\frac{9.1e^{-5s}}{47s+1}$ $\frac{-39e^{-7s}}{181s+1}$ $\frac{-105e^{-4s}}{23s+1}$

4.2 Directionality Properties

The directionality properties of the estimated gain matrices are illustrated in Table 2.

It shows that the directionality properties of the gain matrix estimated from a simple step test are surprisingly close to those of the true system. The multi-sinusoidal experiments did not perform well in this respect, especially not the one with directional excitation. There is not much difference between uncorrelated and directional PRBS excitation.

Table 2. Directionality properties of estimated gain matrices.

Exp.	Uncorrelated				Directional			
	σ_i	V			σ_i	V		
Step DRP	108	–0.036	0.156	0.987	126	–0.039	0.184	0.982
	76.3	0.574	0.812	–0.108	70.0	0.672	0.732	–0.111
	2.33	0.818	–0.563	0.119	4.60	0.739	–0.656	0.152
PRBS	120	–0.034	0.169	0.985	131	–0.041	0.164	0.986
	64.9	0.662	0.742	–0.105	62.4	0.633	0.767	–0.101
	4.15	0.749	–0.648	0.137	4.47	0.773	–0.620	0.135
MSS	109	–0.055	0.148	0.987	131	0.016	0.147	0.989
	87.5	0.417	0.902	–0.112	95.0	0.861	0.501	–0.088
	1.24	0.907	–0.406	0.112	2.05	0.509	–0.853	0.119

4.3 Cross Validations

Any measure of the model fit with the experimental data has not been presented, because such a measure is not very useful. If the excitation of the system is not adequate, it tends to be easier to fit a model well to the data than if the system had been more heavily excited. Even if the fit is good, the obtained model is not likely to be a good representation of the system in such a case.

A better quantification of the model quality is obtained by cross validations. The idea is to check the performance of the model on other data than the data to which the model was fitted. In this case, it is convenient to use data from the other identification experiments to validate a model.

Table 3 shows average prediction errors of the various models for each output y_i, and overall, when applied to all identification experiments. The average prediction errors of the true system has also been included.

Table 3. Average prediction errors (%) by cross validation.

Model	y_1	y_2	y_3	Overall
True	15.11	15.93	14.31	15.12
Step	16.09	17.19	15.35	16.21
DRP dir.	16.50	20.83	17.11	18.15
PRBS unc.	15.96	16.12	16.77	16.28
PRBS dir.	16.63	18.23	20.37	18.41
MSS unc.	16.88	18.11	16.43	17.14
MSS dir.	17.23	20.79	14.68	17.56

It is surely surprising that the average prediction errors of the true model are so large. This reflects the severity of the included disturbances although they are hardly noticed in the outputs. The average prediction errors of all estimated models are quite close to the prediction errors of the true model. Thus, the models should probably not be ranked based on this cross validation. It should also be noted that this cross validation is not a control-oriented validation.

5 CONCLUSIONS

The design of experiments for identification of multivariable systems has been studied. Design guidelines for inputs such as double rectangular pulses (DRP), pseudo-random binary sequences (PRBS), and multi-sinusoidal signals (MSS) were given. These can be applied directly to the system inputs, or they can be used to explicitly excite the various gain directions of a system. By doing that, reliable information about small gains and fast dynamics are easier to obtain than by traditional methods. The low-gain properties are important when a model is used for controller design, less so if the model is used for simulation. In this study, the estimated models were not tested for their control-oriented properties.

6 REFERENCES

/1/ Isermann R., Münchhof M.: Identification of Dynamic Systems. Springer, Berlin and Heidelberg, 2011.

/2/ Ljung L.: System Identification: Theory for the User. Prentice Hall, Upper Saddle River, NJ, 1999.

/3/ Koung C.-W., MacGregor J.F.: Design of identification experiments for robust control. A geometric approach for bivariate processes. Ind. Eng. Chem. Res., 32 (1993) 8, 1658–1666.

/4/ Häggblom K.E., Böling J.M.: Experimental evaluation of input designs for multiple-input multiple-output open-loop system identification. Proc. IASTED Int. Conf. Control and Applications, Honolulu, HI, USA, 2013, pp. 157–163.

/5/ Ghosh R., Häggblom K.E., Böling J.M.: Control-relevant input excitation for system identification of ill-conditioned $n \times n$ systems with $n > 2$. Proc. 19th IFAC World Congress, Cape Town, South Africa, 2014, pp. 9382–9387.

/6/ Garcia S.E., Morari M.: Internal model control. 2. Design procedure for multivariable systems. Ind. Eng. Chem. Process Des. Dev., 24 (1985) 2, 472–484.

/7/ Skogestad S., Morari M., Doyle J.C.: Robust control of ill-conditioned plants: High-purity distillation. IEEE Trans. Autom. Control, 33 (1988) 12, 1092–1105.

/8/ Häggblom K.E.: On experiment design for identification of ill-conditioned systems. Proc. 19th IFAC World Congress, Cape Town, South Africa, 2014, pp. 1428–1433.

/9/ Ljung L.: System Identification Toolbox™ User's Guide. The MathWorks, Inc., Natick, MA, 2014.

/10/ Pintelon R., Schoukens, J.: System Identification: A Frequency Domain Approach. Wiley, Hoboken, NJ., 2012.

/11/ Vasnani V.U.: Towards relay feedback auto-tuning of multi-loop systems. PhD Thesis, National University of Singapore, 1994.