IDENTIFICATION AND MODEL STRUCTURES: POLE-PLACEMENT VERSUS PI- AND PID CONTROL

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Abstract. A parametric model for a laboratory scale ventilation system is identified. Based on the model, digital controllers are developed using a standard structure for pole-placement design. The performance of the controllers is shown to be crucially dependent on the number of parameters in the identified model: Even though no significant difference can be perceived in the ability of the identified models to describe the underlying system, i.e., no apparent model mismatch in control-relevant frequencies, designed controllers will either perform well based on a statistically worse model or fail based on a better model. The pole-placed controllers are further compared to digital PI and PID-control. The implications of the comparative study for the entire control design process are discussed.

1. Introduction

In an earlier study [WW15], two digital pole-placed controllers were designed, compared and evaluated. In this study, the comparison is extended to digital PI- and PID-controllers. In brief, a linear, discrete-time model is identified and digital controllers are designed based on the identified model. Pole-placed controllers are compared to PI- and PID- controllers by analyzing and evaluating the performance of the closed loop systems.

For pole-placed controllers, it seems that the performance of the resulting control system is highly dependent on the number of parameters used in the identified model. Indeed, identified models that by traditional measures are of poorer quality are found to be the only useful models for a successful practical implementation of the closed-loop systems. Furthermore, these results appear not to be the result of model mismatch in a control-relevant frequency region. This observation is contradictory to a common belief within control engineering, i.e., that a model has to be accurate enough. This, often takes the form of a question, e.g., "how accurate the model needs to be for a successful control design" as stated in [ÅW97]. Based on controller comparisons for the case study, several crucial questions in terms of practical identification for control and the entire control design process are raised. For the PI- and PID-controllers, on the other hand, there is apparently no such model sensitivity.

2. Description of the process

For the case study, a laboratory scale ventilation process is studied. The measurements considered are the flow of air, y, and the actuator signal to the fan, u. A step experiment is illustrated in Fig. 1 and simulated step responses are included. The simulations are made using continuous model with a single time constant to describe the dynamics: The initial response is described well with $T = 1.2$ seconds while $T = 0.6$ seconds better describes the latter part of the step response. A parallel system with two different time constants might yield better agreement with the step

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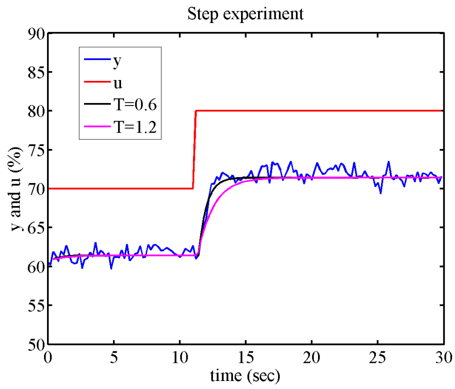

FIGURE 1. Step experiment, flow, y in blue, actuator signal, u in red. Simulated step responses using a single time constant, $T = 0.6$ in black and $T = 1.2$ in magenta, are included. All signals are provided in %.

response, but still the process is quite simple and should therefore be well suited for different control strategies. In the current paper, digital controllers by pole-placement design are considered and compared to digital PI and PID controllers. From the experiment, it can be seen that the measurements are quite noisy, which might be important to consider for a successful design of the controller.

3. IDENTIFICATION EXPERIMENT AND IDENTIFIED MODEL

The considered controllers will be based on a linear model of the process. For this reason, a random binary signal is used for the identification experiment. The input switches between 60% and 80%, yielding an output of around 60%. The sampling period, T_s, is 0.2 seconds with a switching time of $4T_s$, yielding some 4500 measurements for the 15 minute experiment. The proper selection of a sampling period is important in terms of computer-controlled systems and suggestions based on the open-loop process as well on the desired closed-loop behavior are available in [ÅW97]. With respect to the open-loop process and a step experiment similar to the one illustrated in Fig. 1, it is suggested to use a sampling period between $T/10$ and $T/4$, where T is the theoretical time constant for the first order system. For this case study, an approximate theoretical time constant is between 0.6 and 1.2. Thus a sampling period between 0.06 and 0.3 seems appropriate and the choice of $T_s = 0.2$ is motivated. The choice of sampling period from a closed-loop perspective is addressed in Section 5. A segment of the identification experiment is illustrated in Fig. 2. For the

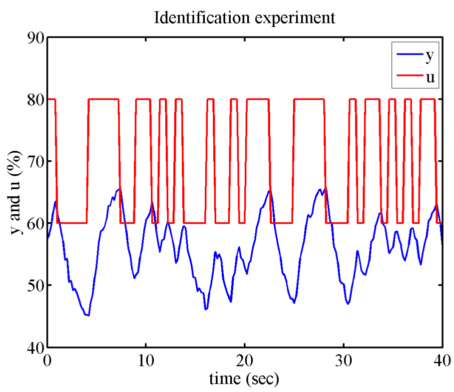

FIGURE 2. Identification experiment, flow, y in %, in blue, actuator signal, u in %, in red.

sake of simplicity and for numerical reasons, all models and controllers are reported with mean values for y and u removed.

A standard ARX-structure is selected for the model to be identified and the model will thus have the form

$$H(z) = \frac{B(z)}{A(z)} \tag{1}$$

where B and A are polynomials in z^{-1}. Based on the identification experiment, the choice of high order models is motivated by statistical criteria, i.e., Rissanen's descripiton length [Ris80] and Akaike Information Criterion [Aka74], as well as by the use of separate sets for estimation and model selection. On the other hand, a simple model might be adequate based on an inspection of the step experiment illustrated in Fig. 1. Indeed, it will be seen that, for digital pole-placed controllers, a simple model with only four parameters will result in an efficient control system, while controllers based on models with more parameters will fail. This observation is one of the main topics of the current paper: The fundamental question from an identification for control point of view is *not* when the model is a sufficient description. Rather, a model that by traditional measures is inferior turns out to be useful, while a more accurate model is not. To illustrate this point, two similar models are considered and used to illustrate the sensitivity of the closed-loop systems for the number of parameters in the models

On the other hand, the PI- and PID-controllers do not appear to be sensitive for these minor differences in the model. The differences between digital pole-placed controllers and PI- and PID-controllers thus also form a basis for a more general discussion on the importance of factors not explicitly considered by the method used for control design.

The first model uses four parameters:

$$H_1(z) = \frac{0.090z^{-1} + 0.065z^{-2}}{1 - 0.77z^{-1} + 0.044z^{-2}} \tag{2}$$

and will be referred to as the first model. The model with five parameters,

$$H_2(z) = \frac{0.097z^{-1} + 0.052z^{-2}}{1 - 0.73z^{-1} - 0.23z^{-2} + 0.15z^{-3}} \tag{3}$$

will be referred to as the second model. The main differences are, as expected, between the coefficients in the A-polynomial and can clearly be seen in pole-zero plot: Both models share a pole in ≈ 0.80, but in addition, the first model has a pole approximately at the origin while the second model has two poles in ≈ 0.4 and ≈ -0.4. The frequency response for the two models are illustrated in Fig. 3.

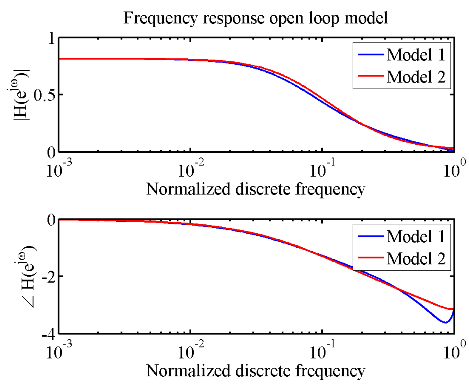

FIGURE 3. Upper panel: $|H_1(e^{j\omega})|$ in blue and $|H_2(e^{j\omega})|$ in red. Lower panel: $\angle H_1(e^{j\omega})$ in blue and $\angle H_2(e^{j\omega})$ in red

As the figure reveals, the models are very similar and the differences are limited to the phase shift for higher frequencies, i.e., very close to the Nyquist frequency. This difference should not, however, be of significant importance for designing the control systems.

The step responses for the models are even more similar as illustrated in Fig. 4.

4. Designing pole-placed controllers

In this section, a (standard) structure for design of digital controllers by pole-placement is considered. For this system, the choice of design method for the controller is somewhat arbitrary:

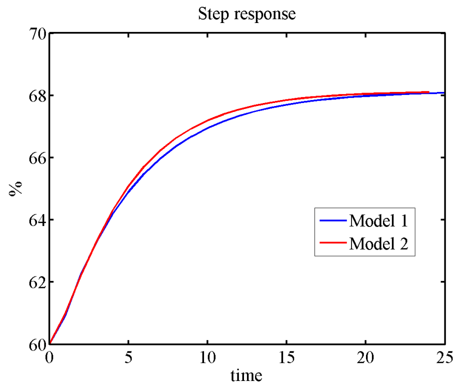

FIGURE 4. Step response for the open loop systems following a change in u from 70% to 80%. Model 1 in blue andModel 2 in red.

No prior specifications are provided and the quality of the control is considered in terms of the need to follow a changing reference signal with a reasonable rise-time, small overshoot and moderate actuator signal activity. In addition, the system should compensate for load disturbances. Such specifications of control criteria are not uncommon, and the choice of an appropriate design methods is not simple since most methods "focus on one or two aspects of the [control] problem and the control-system designer then has to check that the other requirements are also satisfied" [ÅW97]. The design of PI- and PID-controllers that meet similar criteria are presented in the next section.

For the present case study, the choice of design method by pole-placement for the controller can also be motivated by the parametric model identified in addition to the control criteria stated above. This type of controller has also been found to perform well in comparison to other strategies [CL94, TWD11, EVGP14]. Furthermore, the design provides a useful base for adaptive control [ÅW95].

The block diagram for the closed-loop system is illustrated in Fig. 5. The controller is given

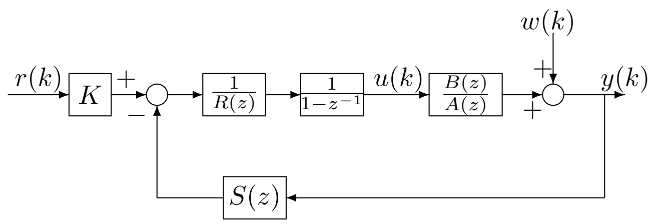

FIGURE 5. Block diagram for the closed-loop system for controlling the process $B(z)/A(z)$. The digital controller includes explicit integral action.

by the gain K, the filter $S(z)$ and the filter $R(z)$ with explicit integral action through the block $1/(1 - z^{-1})$ in order to compensate for load disturbances. The transfer function from the reference signal r to the output y is given by

$$H_{ry}(z) = \frac{KB(z)}{A(z)(1 - z^{-1})R(z) + S(z)B(z)} \tag{4}$$

The polynomials $R(z)$ and $S(z)$ are determined by choosing the poles of $P(z) = A(z)(1-z^{-1})R(z)+S(z)B(z)$. Typical choices are poles at the origin which results in dead-beat control and a varying number of poles on the real axis between 0 and approximately 0.8. For the case study, the final decision is based on simulated step changes in the reference signal and corresponding changes in output y and actuator signal u. Some simulations are illustrated in Fig. 6. The figures illustrate three different choices for poles: The two upper plots use one pole in 0.7, the middle plots have two poles in 0.7 and the lower plots have three poles in 0.7. As the figure reveals, no significant differences can be seen between the closed-loop systems, based on the first and second model, respectively.

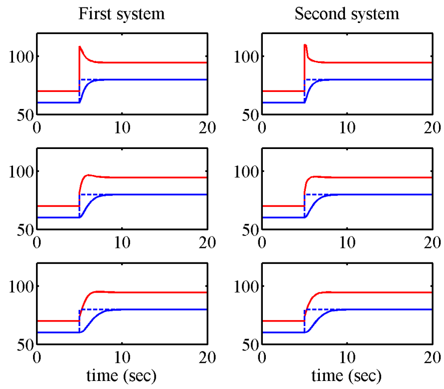

FIGURE 6. Noise-free simulated step changes in r for the control systems based on the first model (left column) and based on the second model (right column). Upper plots use one pole in 0.7, middle plots have two poles in 0.7 and the lower plots have three poles in 0.7. The output y in blue, actuator signal u in red and reference signal r dashed blue, all in %.

The controller is chosen based on a compromise between fast output response and actuator signal activity and the two lower plots thus form good candidates. For this case study, the controllers in the middle plots of the figure are selected. In terms of closed-loop performance, the corresponding continuous transfer function from reference signal to actual value can be approximated by a second order continuous system with damping 0.9 and natural frequency $\omega = 1.5$. [ÅW97] suggest using the natural frequency for the desired closed-loop behavior as a basis for choosing an appropriate sampling period, i.e., T_s between $0.1/\omega$ and $0.6/\omega$. In the present study, $T_s = 0.2$ thus seems appropriate with respect to the closed-loop behavior. In Section 3, it was already noted that the sampling period with respect to the open-loop system also follows suggestions provided in [ÅW97].

For the first model, the controller, for brevity called the first controller, is thus given by

$$\begin{aligned} K_1 &= 0.58 \\ R_1(z) &= 1 + 0.13z^{-1} \\ S_1(z) &= 2.64 - 1.97z^{-1} - 0.087z^{-2} \end{aligned} \tag{5}$$

The second controller is given by

$$\begin{aligned} K_2 &= 0.60 \\ R_2(z) &= 1 - 1.12z^{-1} \\ S_2(z) &= 15.0 - 28.1z^{-1} + 16.9z^{-2} - 3.17z^{-3} \end{aligned} \tag{6}$$

The differences in the R- and S-polynomials can be noted.

5. DESIGNING PI- AND PID-CONTROLLERS

The digital PI- and PID-controllers considered are in velocity form, i.e.,

$$u(k) = u(k-1) + K_c\left(\left(1 + \frac{T_s}{T_i} + \frac{T_d}{T_s}\right)e(k) - \left(1 + 2\frac{T_d}{T_s}\right)e(k-1) + \frac{T_d}{T_s}e(k-2)\right) \tag{7}$$

where $e(k) = r(k) - y(k)$ is the control error, K_c is the proportional gain, T_i is the integral time and T_d is the derivative time. The parameters K_c, T_i and T_d are determined by minimizing the sum squared difference between the control errors for the pole-placed and PI-/PID-controller respectively following a simulated filtered step change in the setpoint $r(k)$. For filtering the step change, the first-order filter

$$r_f(k) = (1-\alpha)r(k) + \alpha r_f(k) \tag{8}$$

is used with $\alpha = 0.9$. The controllers thus obtained are given in Table 1.

In the table, the subscripts corresponds to the model and pole-placed controller used in the simulations.

Controller	K_c	T_i	T_d
PI_1	0.714	0.694	
PI_2	0.660	0.640	
PID_1	0.722	0.715	-0.162
PID_2	0.646	0.622	-0.000455

TABLE 1. PI- and PID-parameters determined based on closed-loop simulations of Model 1 and Model 2 respectively.

6. Evaluating the control systems

Once the controllers have been designed, a multitude of options for evaluating the closed-loop systems is available.

For the case study, the controllers were implemented on the actual system. The performances of the pole-placed controllers are illustrated in Figs. 7–8. As the figures reveal, the first controller

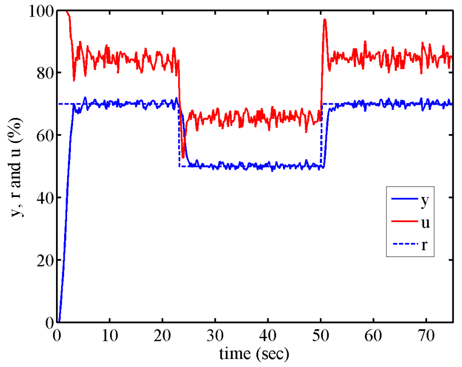

FIGURE 7. Control of the ventilation system for a few step changes using the first controller. The output y in blue, actuator signal u in red and reference signal r dashed blue.

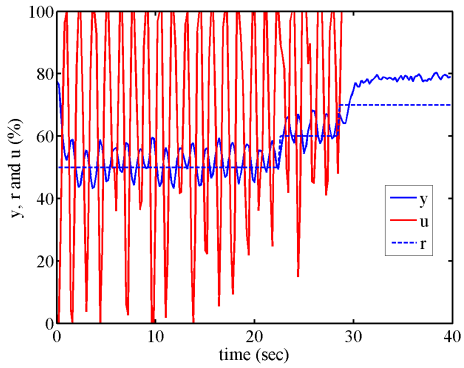

FIGURE 8. Same as in Fig. 7 but using the second controller.

performs acceptably. In the beginning of Fig. 7 it can be seen how the system is started and handled very well by the controller. The second controller, on the other hand, has very poor performance and collapses when a higher setpoint is attempted. In practical trials, the second closed-loop system often stalls with the control signal at either 0 or 100% and will not show the oscillations of Fig. 8.

Here, practical evaluations of PI- and PID-controllers will be included in the final manuscript.

6.1. **Analyzing the pole-placed controllers.**

In order to explain the significant and perhaps surprising differences between to two systems with pole-placed controllers, a natural choice would be to investigate the sensitivity of the closed-loop system to modeling errors since this is an important aspect in all design work. Indeed, much work in identification for control has been focused on model uncertainty as presented in [Gev05]. One way of investigating the sensitivity of the system to model uncertainties is to study the sensitivity function. For the closed-loop system of Fig. 5, this corresponds to the transfer function from measurement noise w to output y given by

$$H_{wy}(z) = \frac{A(z)(1 - z^{-1})R(z)}{A(z)(1 - z^{-1})R(z) + S(z)B(z)} \tag{9}$$

For the two systems with pole-placed controllers, the corresponding amplitudes as a function of frequency, $|H_{wy}(e^{j\omega})|$, are illustrated in Fig. 9. As can be seen, the second system is somewhat

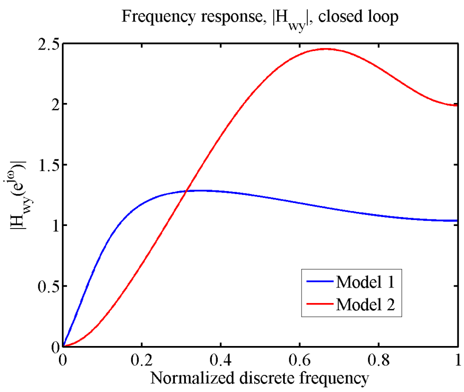

FIGURE 9. $|H_{wy}(e^{j\omega})|$ based on the first model in blue and based on the second model in red.

worse and does not quite meet the suggested requirement for reasonable robustness against instability, i.e., $|H_{wy}(e^{j\omega})| < 2$ [ÅW97]. Still, it is doubtful whether the differences between $|H_{wy}(e^{j\omega})|$ for the two systems can fully explain the results of Figs. 7–8. Another possible explanation is that the nonlinearity due to actuator signal saturation is not considered in the model. Although a matter of some concern, the control attempted Fig. 8 should not require such large control signals. Therefore, other reasons for the failure of the second controller must be considered.

Given the significant measurement noise, it seems motivated to evaluate noisy simulations. As can be seen in Figs. 10–11, the difference between the two systems is drastic when noise is included. Notice the different scales used for the y-axis in the two figures.

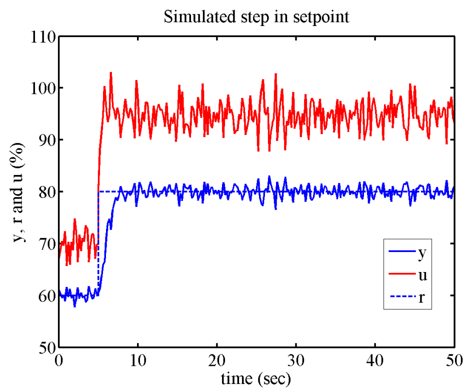

FIGURE 10. Simulated step change in r for the control systems based on the first model including measurement noise with unit variance. The output y in blue, actuator signal u in red and reference signal r dashed blue.

An explanation for this critical difference can be seen in the transfer function from measurement noise to actuator signal. For the closed-loop system of Fig. 5, this is given by

$$H_{wu}(z) = -\frac{A(z)S(z)}{A(z)(1 - z^{-1})R(z) + S(z)B(z)} \tag{10}$$

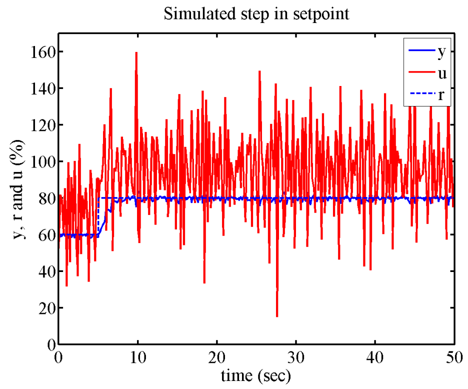

FIGURE 11. Same as in Fig. 10 but for the control systems based on the second model.

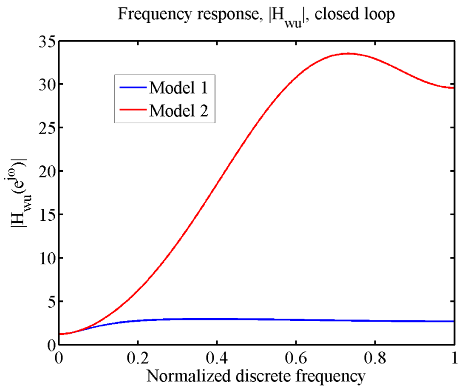

FIGURE 12. $|H_{wu}(e^{j\omega})|$ based on the first model in blue and based on the second model in red.

The corresponding amplitudes as a function of frequency, $|H_{wu}(e^{j\omega})|$, based on the first and second controllers are illustrated in Fig. 12.

As the figure clearly illustrates, the actuator signal activity for the closed-loop system based on the second model will, in practice, render the control system useless, i.e., high frequency measurement noise with a variation of $\pm 1\%$ in open loop will render control signals in excess of $\pm 25\%$. The corresponding practical failure is illustrated in Fig. 8. A preliminary study of possible reasons for this failure is provided in [WW15], but for solid explanations further study is required. Here, the focus is on a comparison between pole-placed and PI-/PID-controllers.

6.2. **Analyzing the PI- and PID-controllers.** A similar analysis for the PI- and PID-controllers yields

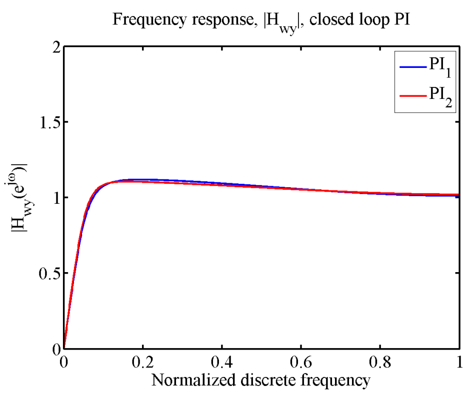

FIGURE 13. Same as in Fig. 9 but for the PI-controllers.

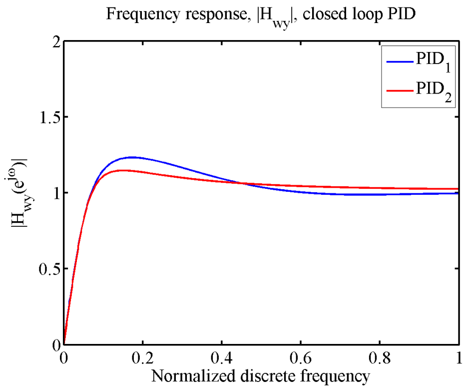

FIGURE 14. Same as in Fig. 9 but for the PID-controllers.

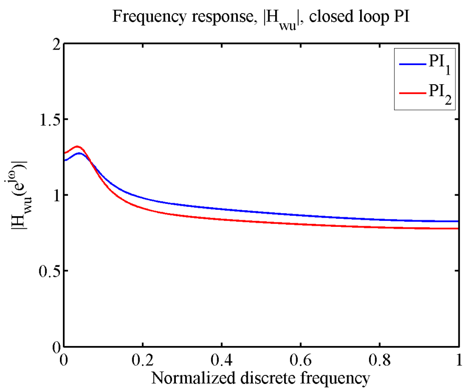

FIGURE 15. Same as in Fig. 12but with the PI- controllers.

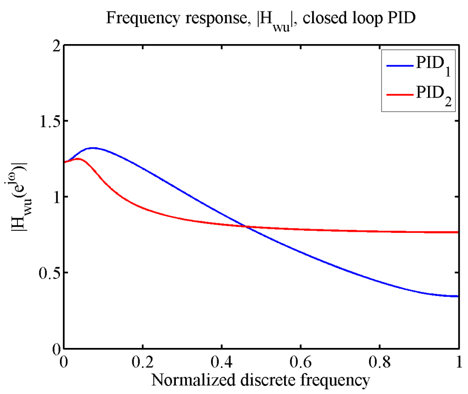

FIGURE 16. Same as in Fig. 12but with the PID- controllers.

6.3. **Comparing the approaches for control design.** In the final manuscript, a more detailed comparison with explanatory figures will be included. In summary, the (working) pole-placed controller is slightly faster and gives slightly better low-frequency damping of disturbances. For the case study, the pole-placed controllers displayed a risk for a high gain from disturbances to control signal not present in the PI- and PID-controllers. For this reason, the PI- and PID-controllers seem better suited for the application studied.

7. THE CASE STUDY—IMPLICATIONS

The case study illustrates the entire control design process starting with an identification experiment and resulting in an implementation of the controller. Depending on the choice of design method, it was seen that the number of parameters chosen for the identified model can be crucial in terms of whether a working solution is obtained or not. Perhaps counter-intuitively, an identified model that by traditional measures is inferior was found to be the only useful model for a successful practical implementation of the closed-loop system using pole-placed controllers. Valuable insights may be obtained by investigating the reasons for and the implications of such a drastic difference. For the PI and PID-controllers, however, no such sensitivity could be observed.

Even though the present case study is limited to the identification of an ARX-type model and two different strategies for digital control, the study has some general implications for control system design. Specifically, one of the five future challenges facing the field of control recognized by the panel for future directions in control is formulated as "Automatic synthesis of control algorithms . . . [and] more powerful design tools that automate the entire control design process from model development to hardware-in-the-loop simulation" [MÅB+03]. An important step for tackling this challenge could be developing tools that allow control-system designers to choose different control strategies and visually inspect aspects not explicitly addressed by the chosen strategy. For example, in the present case study, the choice of the design parameters for the pole-placed controllers, i.e., the poles of the closed-loop system, quite explicitly determines the rise-time, overshoot and actuator signal activity due to changes in reference signal. Clearly, equal consideration for actuator signal activity due to measurement noise was required for practical success, although this factor was not explicitly addressed by the chosen method of control design. In addition, other factors, such as robustness against variations and uncertainties in process behavior may be of significance. The PI- and PID-controllers were simply designed for similarity to the pole-placed controllers without explicitly addressing any aspects of control design. The resulting controllers, however, were seen to perform well with respect to all studied criteria. In this respect, the PI- and PID-controllers are a more reliable choice for this case study.

These observations, however, reveal an apparent need for data-based methods for selecting control strategy. Such methods could be of great importance for automating the entire control design process.

Developing such tools or methods is not, however, an easy task as illustrated by the following brief summary of the choices within identification and control design.

(1) Identification. For the case of no prior model and no existing control system a model needs to be identified based on an open loop experiment. Some interrelated questions are: What sampling period is appropriate? What type of and what levels for the excitation signal should be used? Should a linear or nonlinear model be considered? What type of model structure, e.g., ARX or ARMAX within the parametric linear category, should be used? How many parameters within the selected model structure should be used? The case study clearly showed the great significance even of this last detail.
(2) Control strategy. When choosing a suitable control strategy, an array of factors should be considered, e.g., goals for the control system, attenuation of load disturbances, reduction of the effect of measurement noise, variations and uncertainties in process behavior, actuator signal activity, the types of models available, suitable design parameters for the chosen control strategy, etc. Even if no control strategy explicitly addresses all factors, other factors must still be considered as is clearly illustrated in the present article.

Furthermore, identification is clearly linked to control strategy with respect to the type of model, information deemed valuable, etc. These observations can thus be summarized by the following two, perhaps alternative, questions:

- Can an identification experiment provide guidelines for a suitable choice of controller strategy?
- Keeping in mind the goal of an efficient closed-loop system, can the choice of control strategy be used to determine excitation signal, model structure and model selection in the identification of the process?

In an attempt to further clarify these questions, the case study will be considered in future work on a more extensive comparative study of different methods for control system design.

8. Conclusions

A case study for the control design process starting from an identification experiment and resulting in an efficient implementation of a controller was described. It was seen that the resulting control system was extremely sensitive to the number of parameters in the identified model: An apparent arbitrary choice of four or five parameters had little apparent influence on the identified model, but meant the difference between an efficient closed-loop system and a complete failure. The reasons for this surprising observation were analyzed, showing a clear need for useful tools to account for factors not explicitly considered by the method used for control design. Alternatively, experimental methods indicative of suitable control strategies and corresponding schemes for identification and model structure selection would be of great practical value.

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