Experiment Designs for Identification of Multivariable Dynamic Systems

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ABSTRACT

Systems with multiple inputs and multiple (MIMO) outputs are more difficult to identify than systems with a single input and a single output (SISO) due to interactions between the variables. Surprisingly little is said about the experiment design for MIMO system identification in major textbooks on system identification. The most advanced advice is that the inputs should be perturbed simultaneously in an uncorrelated way. It has been noted that this kind of excitation tends to produce poor data for system identification if the model is to be used for control system design. In this paper, an overview is given of proposed methods to overcome this problem by excitation of the (estimated) gain directions of the system. A new method, where the dynamics are explicitly taken into account, is also presented. This method is significantly simpler than some other methods proposed for inclusion of dynamics in the design. Thus, it is suitable for practical application. The methods are illustrated and compared by an instructive example of a 2×2 system. A dynamic experiment design for a 3×3 system is also presented.

1 INTRODUCTION

System identification is the art and science of determining a mathematical model from data that describe the behaviour of the system. An essential part of this endeavour is the experiment design for generating the data. This, in turn, should be a function of the intended use of the model. For example, if the model is to be used for design of a feedback controller, closed-loop properties are important. The numerical algorithms used for determining the model cannot produce a good model if the data is a poor representation of the system for the intended use.

Systems with multiple inputs and multiple outputs (MIMO) are generally more difficult to identify than systems with a single input and a single output (SISO) due to interactions between the variables. Because of the interactions, controllability issues are more difficult to handle in MIMO systems than in SISO systems. Surprisingly little is said about experiment design for MIMO system identification in major textbooks on system identification /11, 13, 15/. The most advanced advice is that the inputs should be perturbed simultaneously in an uncorrelated way using, for example, uncorrelated pseudo random binary sequences (PRBS) that mimic the properties of white noise. However, this design might result in strongly correlated outputs, which is not good for identifiability.

In the control of industrial MIMO systems, integral controllability (IC) is usually a required property. This means that the closed-loop system must be stable when controllers with integral action are used, and remain stable if the controllers are detuned /1, 12/. This motivated an experiment design based on an estimate of the static gain matrix of the system /12/. In this design, the static gain directions of the system are explicitly excited. An overview of various developments of this procedure is given in /2/.

It is desired to identify the dynamics of the system although the design is based on the static gain matrix. In practice, this often works well /7, 8/. However, a systematic handling of input and output constraints is difficult in the procedure. Furthermore, the method may, due to the dynamics of the system, result in correlated outputs similarly to the case when uncorrelated inputs are used. To overcome these drawbacks of the gain-directional design, a new design procedure has been proposed /5, 6/. In this procedure, the correlation of the outputs is minimized based on an estimate of a dynamic model of the system.

In this paper, an overview of the gain-directional design as well as the new optimization-based dynamic design is given. It is shown that the dynamic design can be implemented in the same simple way as the gain-directional design. Thus, it is very suitable for practical application. The methods are illustrated and compared by an instructive example of a 2×2 system with two inputs and two outputs. A dynamic experiment design for a 3×3 system, for which gain-directional designs have previously been done /3, 4, 9/, is also presented.

2 EXPERIMENT DESIGN

2.1 Gain-Directional Design

Consider a system with the transfer function matrix $G(s)$, the input vector $u(t)$, and the output vector $y(t)$, where the argument t denotes time. Let K be an estimate of the static gain matrix $G(0)$. Define

$$\bar{y}(t) := Ku(t), \tag{1}$$

where $\bar{y}(t)$ is the output a static system would have. A singular value decomposition (SVD) of K yields

$$K = W\Sigma V^{\mathrm{T}}, \tag{2}$$

where W and V are orthogonal matrices and Σ is a diagonal matrix of singular values. The orthogonality means that $V^{\mathrm{T}}V = I$ and $W^{\mathrm{T}}W = I$, where I is an identity matrix of appropriate dimension.

A new signal vector, the design signal $\xi(t)$, is introduced. The design signal has n vector components, where n is the number of singular values in Σ. It is implemented by the input

$$u(t) = V\Sigma^{-1}\xi(t). \tag{3}$$

It is instructive to consider the output $\bar{y}(t)$ with this input. Substitution of Eqs (2) and (3) into Eq. (1) yields

$$\bar{y}(t) = W\xi(t) = \sum_{i=1}^{n} w_i \xi_i(t), \tag{4}$$

where w_i is the i^{th} column of W and $\xi_i(t)$ is the i^{th} component of $\xi(t)$. This means that $\xi_i(t)$ excites the i^{th} gain direction of the system. It follows from Eq. (4) that

$$P_{\bar{y}} = WP_{\xi}W^{\mathrm{T}}, \tag{5}$$

where P_x denotes the covariance matrix of a vector-valued time series $x(t)$.

The gain directions can be excited in different ways, e.g., sequentially one at a time or simultaneously, using different kinds of inputs such as step changes (pulses), pseudo random binary sequences (PRBS) and multi-sinusoidal signals /2/. If the gain directions are excited simultaneously, the signals $\xi_i(t)$, $i = 1, \ldots, n$, should be mutually uncorrelated. If they all have the variance a^2, $P_{\xi} = a^2 I$ and $P_{\bar{y}} = a^2 WW^{\mathrm{T}}$. If the system has equally many outputs and inputs, W is square and $WW^{\mathrm{T}} = I$, resulting in $P_{\bar{y}} = a^2 I$. This means that the static outputs $\bar{y}_i$, $i = 1, \ldots, n$, are uncorrelated.

2.2 Static Design with Constraints

Assume that the input $u(t)$ has the covariance matrix P_u. From Eq. (1) it then follows that

$$P_{\bar{y}} = KP_u K^{\mathrm{T}}. \tag{6}$$

If $P_{\bar{y}}$ is defined by the design objective, and K is non-singular, P_u can be solved out as

$$P_u = K^{-1} P_{\bar{y}} K^{-\mathrm{T}}, \tag{7}$$

where $K^{-\mathrm{T}}$ denotes the inverse of K^{T}. If $P_{\bar{y}}$ is specified as in Eq. (5),

$$P_u = K^{-1} W P_{\xi} W^{\mathrm{T}} K^{-\mathrm{T}} = V \Sigma^{-1} P_{\xi} \Sigma^{-1} V^{\mathrm{T}}. \tag{8}$$

If the design signal has the property $P_{\xi} = a^2 I$, $\Sigma^{-1} P_{\xi} \Sigma^{-1}$ is diagonal, from which it follows that P_u has the SVD

$$P_u = V \Lambda V^{\mathrm{T}}, \quad \Lambda = \Sigma^{-1} P_{\xi} \Sigma^{-1}. \tag{9}$$

This means that the gain-directional design can be obtained from a SVD of P_u as

$$u(t) = V \Lambda^{1/2} \xi(t), \tag{10}$$

where $\Lambda^{1/2}$ replaces Σ^{-1} in Eq. (3).

The advantage of using Eq. (6) for obtaining P_u, and subsequently the input design (10), is that variance constraints on $y(t)$ and $u(t)$ can be handled. Assume that the constraints are

$$P_{u,ii} \le R_{u,ii}, \quad P_{\bar{y},ii} \le R_{\bar{y},ii}, \tag{11}$$

where $R_{u,ii}$ and $R_{\bar{y},ii}$ denote the maximum allowed variances of u_i and $\bar{y}_i$, respectively. It is desired to obtain uncorrelated outputs, which means that $P_{\bar{y}} = R_{\bar{y}}$ is desired if permitted by $P_{u,ii} \le R_{u,ii}$. According to Hadamard's inequality, for any positive definite matrix P_x, $\det P_x \le \prod_{\forall i} P_{x,ii}$, with equality if and only if P_x is diagonal /10/. This means that maximizing $\det P_{\bar{y}}$ with respect to P_u, subject to Eqs (6) and (11), will result in $P_{\bar{y}} = R_{\bar{y}}$ if the input constraints are not restrictive. This can be expressed as a convex optimization problem, which is easy to solve using, e.g., MATLAB and the YALMIP software /14/.

This way of handling constraints in the framework of gain-directional input design, is significantly simpler than previously proposed methods (see /2/).

2.3 Dynamic Design with Constraints

Consider a discrete-time state-space model

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) \\ y(k) &= Cx(k) + Du(k) \end{aligned}, \tag{12}$$

where k denotes discrete time expressed as sample index, $u(k)$ is a vector of n_u inputs, $x(k)$ is a vector of n_x state variables, and $y(k)$ is a vector of n_y outputs, all at time instant k. The matrices A, B, C and D are standard state-space matrices of appropriate dimensions. Assume that $x(k)$ and $u(k)$ are uncorrelated. Then,

$$\begin{aligned} P_x &= A_{\mathrm{d}} P_x A_{\mathrm{d}}^{\mathrm{T}} + B_{\mathrm{d}} P_u B_{\mathrm{d}}^{\mathrm{T}} \\ P_y &= C P_x C^{\mathrm{T}} + D P_u D^{\mathrm{T}} \end{aligned}, \tag{13}$$

where P_u, P_x and P_y are stationary covariance matrices of $u(k)$, $x(k)$ and $y(k)$, respectively. To obtain a design that produces uncorrelated outputs, $\det P_y$ is maximized with respect to P_u and P_x subject to Eq. (13) and similar constraints as in Eq. (11). A SVD of P_u yields the required matrices to implement the design by Eq. (10). This way of determining and implementing a dynamic experiment design is significantly simpler than some previously proposed ones (see /5/).

According to Eq. (12), $x(k+1)$ is correlated with $u(k)$, and consequently $x(k)$ is correlated with $u(k-1)$. The requirement that $x(k)$ and $u(k)$ be uncorrelated, then requires that $u(k)$ is uncorrelated with $u(k-1)$, which means that the time series produced by $u(k)$ must contain no autocorrelation. This also means that the components $\xi_i(k)$, $i = 1,\ldots,n$, must contain no autocorrelation. A signal type that satisfies this requirement is a random binary signal (RBS). However, in practice it is usually desired to have an input that covers a certain frequency range more uniformly than a RBS. A pseudo random binary sequences (PRBS) is such a signal, but it contains autocorrelation if the sampling interval is smaller than the minimum switching time, as is often the case.

One way of solving the autocorrelation problem with a PRBS signals is to do the design using a sampling interval equal to the minimum switching time even if a smaller sampling interval will be used in the implementation. Another solution, which works in practice, is to initially neglect the autocorrelation issue. It has been observed that a PRBS signal will produce uncorrelated outputs, but with higher variances than the design variances. Since a dynamic model is available for the design, it can be used to determine the output variances produced by the design. If the simulated output variances are $S_{y,ii}$, $i = 1,\ldots,n$, the optimization can then be redone with the output variance constraints $P_{y,ii} \le R_{y,ii} S_{y,ii}^{-1}$.

3 ILLUSTRATION OF INPUT DESIGNS

In this section, the various input designs are illustrated by a simple example. The system is described by the continuous-time state-space model /16/

$$\begin{aligned} \dot{x}(t) &= A_c x(t) + B_c u(t) \\ y(t) &= Cx(t) \end{aligned}, \quad A_c = \begin{bmatrix} -\frac{1}{194} & 0 \\ 0 & -\frac{1}{15} \end{bmatrix}, B_c = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}, C = \begin{bmatrix} \frac{87.8}{194} & \frac{1.4}{15} \\ \frac{108.2}{194} & -\frac{1.4}{15} \end{bmatrix}, \ K = \begin{bmatrix} 87.8 & -86.4 \\ 108.2 & -109.6 \end{bmatrix}. \quad (14)$$

The gain matrix has the condition number $\kappa \approx 142$, which mean that the system is ill-conditioned. For simplicity, units are left out.

In the following designs, PRBS signals are used as inputs $u(k)$ or design signals $\xi(k)$. The signal amplitude is 1, the sequence length is 255 ($= 2^8 - 1$), and the minimum switching time is 4. The sampling interval is 1 time unit. The two PRBS signals for each design are made uncorrelated by time-shifting them by half the sequence length (approximately). Recommendations on PRBS design are given in /11, 13, 15/.

The true model is used in the gain-directional and dynamic designs. This is considered acceptable since the purpose of the study is to illustrate the designs and the main differences between the results. Tests with approximate models yield very similar results. Because the output correlation is the main interest, input amplitudes are not adjusted (e.g., by redoing the optimization) to make output variances similar in the various designs.

3.1 Uncorrelated Inputs

The standard textbook suggestion of using uncorrelated inputs is illustrated first. Figure 1 shows a scatter plot of $y_2(k)$ vs. $y_1(k)$, which illustrates the output distribution obtained by this design. The determinant of the output correlation matrix has the value 0.005.

As can be seen, the output distribution has a very strong directionality, which is a clear disadvantage when a model is determined from data. The result could, e.g., be that the sign of the determinant of the gain matrix of the estimated model is different from that of the true system. In such a case, controllers with integral action that stabilize the model, will not stabilize the true system.

3.2 Gain-Directional Design

The gain-directional design is implemented by Eq. (3), where $\xi(t)$ is the same vector of PRBS signals as $u(t)$ in the uncorrelated design. A SVD of K yields

$$\Sigma^{-1} = \begin{bmatrix} 0.0051 & 0 \\ 0 & 0.7187 \end{bmatrix}, \quad V = \begin{bmatrix} -0.7066 & -0.7077 \\ 0.7077 & -0.7066 \end{bmatrix}. \tag{15}$$

Figure 2 is a scatter plot of $y_2(k)$ vs. $y_1(k)$ for this design. According to the design, static outputs would be uncorrelated, but here there is a clear correlation due to the dynamics of the system. In this case, the output directionality is weaker than and opposite to the directionality of obtained by uncorrelated inputs. The determinant of the output correlation matrix has the value 0.22.

3.3 Dynamic Design

For the dynamic design, the state-space model (14) is converted to a discrete-time state-space model (12). Using the sampling interval 1 and assuming constant inputs between sampling points, the discretization results in

$$A_\mathrm{d} = \begin{bmatrix} 0.9949 & 0 \\ 0 & 0.9355 \end{bmatrix}, \quad B_\mathrm{d} = \begin{bmatrix} 0.9974 & -0.9974 \\ 0 & 0.9674 \end{bmatrix}. \tag{16}$$

Using the output constraint $R_y = I$, but no input constraint, maximization of $\det P_y$ subject to Eq. (13) yields

$$P_y = \begin{bmatrix} 1.00 & 0.00 \\ 0.00 & 1.00 \end{bmatrix}, \; P_u = \begin{bmatrix} 7.9176 & 7.8182 \\ 7.8182 & 7.7389 \end{bmatrix}, \; \Lambda^{1/2} = \begin{bmatrix} 0.0979 & 0 \\ 0 & 3.9556 \end{bmatrix}, \; V = \begin{bmatrix} -0.7031 & -0.7111 \\ 0.7111 & -0.7031 \end{bmatrix}, \tag{17}$$

where $\Lambda^{1/2}$ and V are obtained from a SVD of P_u. The input directions shown by V are almost exactly the same as the input directions of the gain-directional design, but the amplification of the gain directions shown by $\Lambda^{1/2}$ and Σ^{-1} are different.

Figure 3 is a scatter plot of $y_2(k)$ vs. $y_1(k)$ for this design. Now the outputs are almost completely uncorrelated, the determinant of the output correlation matrix having the value 0.98.

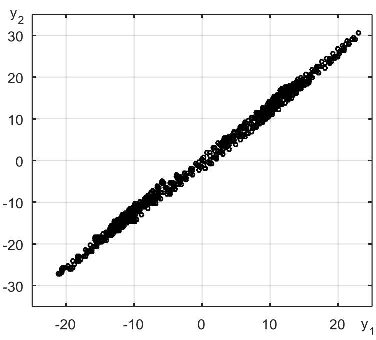

Figure 1. Uncorrelated inputs.

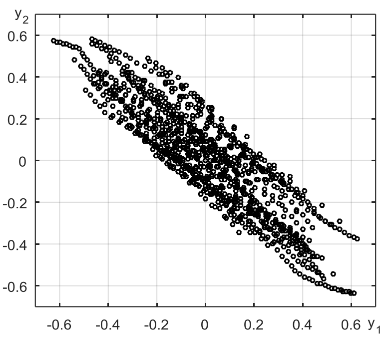

Figure 2. Gain-directional design.

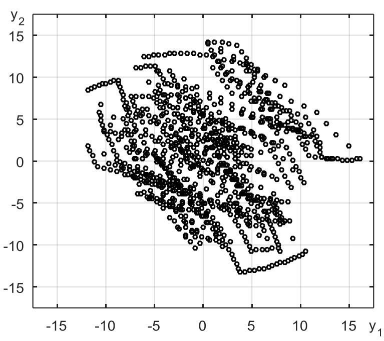

Figure 3. Dynamic design.

4 DYNAMIC DESIGN FOR A 3×3 SYSTEM

The system for this case study has the transfer function

$$G(s)=\begin{bmatrix} \dfrac{6\,e^{-5s}}{22s+1} & \dfrac{20\,e^{-5s}}{337s+1} & \dfrac{-1e^{-5s}}{10s+1} \\ \dfrac{8\,e^{-5s}}{50s+1} & \dfrac{77\,e^{-3s}}{28s+1} & \dfrac{-5\,e^{-5s}}{10s+1} \\ \dfrac{9\,e^{-5s}}{50s+1} & \dfrac{-37\,e^{-5s}}{166s+1} & \dfrac{-103\,e^{-4s}}{23s+1} \end{bmatrix}. \tag{18}$$

The system was originally presented by Vasnani /17/, but here an input and an output have been rescaled to make the system more ill-conditioned and thus more interesting for this study /3, 4, 9/. The condition number of the gain matrix is 30. A SVD of the gain matrix gives the matrices

$$\Sigma^{-1}=\begin{bmatrix} 0.0088 & 0 & 0 \\ 0 & 0.0134 & 0 \\ 0 & 0 & 0.2645 \end{bmatrix},\quad V=\begin{bmatrix} -0.0468 & 0.1575 & 0.9864 \\ 0.5444 & 0.8320 & -0.1070 \\ 0.8375 & -0.5320 & 0.1247 \end{bmatrix}. \tag{19}$$

For the dynamic design, the model is first converted to a discrete-time transfer function using the Z-transform. In this case, the sampling interval 2 is used. Each discrete-time transfer function is converted to a discrete-time state-space model in such a way that the time delays are included as extra state variables in the model. The state-space models for all transfer functions are collected to a single state-space model of the form (12). Because only the time delays in excess of the minimum time delay 3 matter for the input design, the time delay 3 is removed from all transfer functions before the conversion to state-space models. In this way, the order of the full state-space model can be kept as low 16.

Maximization of $\det P_y$ subject to Eq. (13) and the output constraints $P_{y,ii}<1$, $i=1,2,3$, gives the input covariance matrix P_u. A SVD of P_u gives the matrices

$$\Lambda^{1/2}=\begin{bmatrix} 0.0355 & 0 & 0 \\ 0 & 0.0653 & 0 \\ 0 & 0 & 0.8356 \end{bmatrix},\quad V=\begin{bmatrix} -0.0688 & 0.0620 & 0.9957 \\ -0.1360 & 0.9882 & -0.0709 \\ 0.9883 & 0.1403 & 0.0595 \end{bmatrix}, \tag{20}$$

where the signs of columns 2 and 3 in V have been switched to make comparison with (19) easier. The most notable difference is in elements (2,1) and (3,2), which is caused by the very different dynamics in the transfer functions.

Figure 4 shows the inputs and the outputs of the dynamic design using uncorrelated PRBS signals with sequence length 127 and minimum switching time 10 as design signal. Figure 5 shows scatter plots of y_1 vs. y_2, y_1 vs. y_3 and y_2 vs. y_3. As can be seen, the output distribution is well balanced. The determinant of the output correlation matrix is 0.98. In this case, the result of a static gain-directional design is not nearly as good.

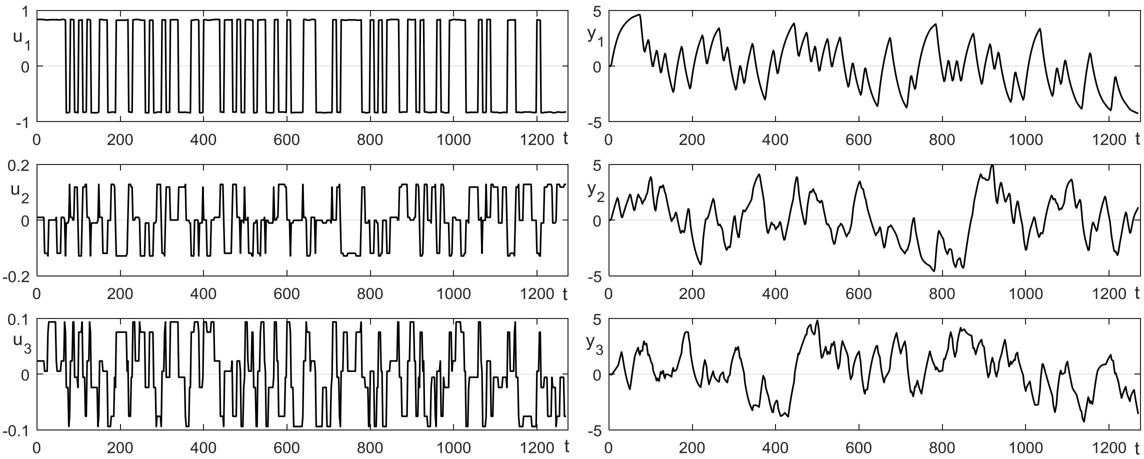

Figure 4. Inputs (left) and outputs (right) of dynamic design.

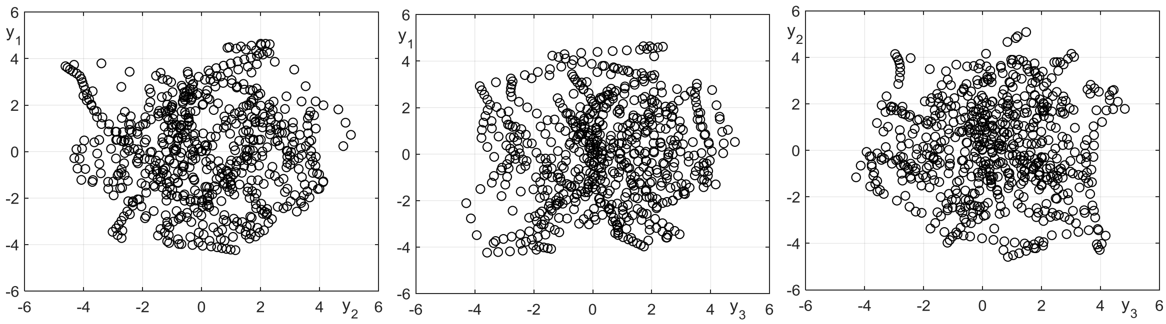

Figure 5. Scatter plots of output distribution of dynamic design.

5 CONCLUSIONS

An overview of experiment design methods for MIMO identification was given with special emphasis on the identification of ill-conditioned systems when the intended use of the model is control system design. A new method, where (estimated) dynamics can be taken into account in the design, was presented. The design method is formulated as a convex optimization problem that can be solved by standard software in a straightforward way. The implementation of the design is simple, which makes the method suitable for practical application.

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