

# Mean-Field Approximation of a Microscopic Population Model for Austria

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## Abstract

Verification and validation of large agent-based models is a complicated process. To check for full functionality, the simulation has to be executed various times, which takes both time and computational resources. In this discussion-paper we present an approach that could generally improve this process, applied on an agent-based population model for Austria. A so-called mean-field model, in this case a partial differential equation (PDE) model is used for this aim. As numeric solution of the PDE only takes a few moments, the mean-field model can provide a fast prospect on results, behaviour and sensitivity of the agent-based model used.

*Keywords:* agent-based modelling, mean-field theory, mean-field approximation, population model, partial differential equations

## 1 Introduction

During the last decades a great number of population models have been developed all over the world (e.g. (Andreassen, 1993; Kelly, 2003; Spielauer, 2013)) which, in combination with economic and/or health models, can be used for supporting internal political decisions. Most of them are microscopic simulation models wherein the population is no longer simulated as a whole (as done in (Malthus, 1798)), but as a sum of individuals.

This type of population modelling has turned out to be advantageous due to its flexibility with respect to model-extensions and hence plays an important role in the Comet founded healthcare project DEXHELPP. In the course of this project an agent-based model (ABM) for Austria's population was developed and implemented (Bicher et al., 2015) in order to create a solid basis for generating decision-support models for Austria's health care system.

## 2 An Agent-Based Population Model for Austria

The basic rules for the mentioned agent-based population model are very intuitive. Each agent is assigned

- age (equivalent to a certain birth-date) and
- sex.

The model is simulated with equidistant time-steps of arbitrary length  $dt$ . With a certain age and time-step-length dependent probabilities each agent might die, emigrate or, in the case of female agents, reproduce. Additionally a certain number of immigrants is added at each step of the model. Thus the model requires the following parameters:

- initial population size [1],
- initial population distribution with respect to age and sex [2],
- immigration age/sex distribution for each simulated time-step [3],
- death [4], emigration [5] and reproduction [6] probabilities with respect to age, sex and time-step and finally
- natural percentage for a male/female offspring [7].

These seven parameters were successfully computed with requested data from the Austrian Bureau of Statistics (Austria, 2012). The model was implemented in Python 3.

Unfortunately, the simulation executed with CPython3.4 is quite slow. As a single simulation with 8 million agents and a timespan of 50 years with daily time steps takes about 10 hours, the verification and validation process turned out to be challenging. To support, mean-field analysis was used.

## 3 Mean-Field Analysis

### 3.1 General Concept

The so-called mean-field poses the most applicable basis to analyse a microscopic model on the macroscopic level. Let  $a_i(t), i = 1 \dots N$  denote the abstract state of agent  $i$  at time  $t$  then the mean-field  $O(x, t) = \sum_{i=1}^N \mathbb{1}_x(a_i(t))$  simply counts all agents in state  $x$  at a given time  $t$ .

In terms of the defined population model the mean-field corresponds to the aggregated numbers: "total population with age  $a$  and sex  $s$  at time  $t$ ". As age is a continuous variable and the model is stochastic, the mean-field poses for a density function of a stochastic process.

The mean field of a microscopic model can have several useful properties and can even be analysed with analytical methods. One method is Mean-Field Approximation, which allows approximation of the mean value of the (stochastic) mean-field by a closed, aggregated equation based-model – a so called mean-field model. Usually a ordinary differential equation (Bicher and Popper, 2013) or difference equation (Gast and Gaujal, 2011) model results, but if states of agents are continuous (as in our case) the method results in a partial-differential equation (PDE) model (Deffuant et al., 2000).

Key to find this mean-field model is applying a so-called Mean-Field Theorem (Tainaka, 1989), a statement closely related to the famous Kolmogorow equations for Markov processes. If applied correctly, this theorem provides information how stochastic analysis of the single agents in the ABM results in an aggregated model for the mean-field.

### 3.2 Application

In order to apply the cited mean-field theorem, the stochastic processes of the single agents need to be analysed. This process is quite long and technical. Hence we refer to (Deffuant et al., 2000) to get an idea about how this can be done.

Let  $F(a, t)$  and  $M(a, t)$  denote the density of female and male  $a$ -year old individuals in Austria at time  $t$ , then the following model describes their dynamics:

**Corollary 3.1** (Mean-Field Model for the Agent-Based Population Model).

$$\frac{\partial M}{\partial t} - \frac{\partial M}{\partial a} = \alpha_1 + \gamma_1 \mathbb{1}_{[0, dt)}(a) \Psi(a, t) - M \delta_1 \quad (1)$$

$$\frac{\partial F}{\partial t} - \frac{\partial F}{\partial a} = \alpha_1 + \gamma_2 \mathbb{1}_{[0, dt)}(a) \Psi(a, t) - F \delta_2 \quad (2)$$

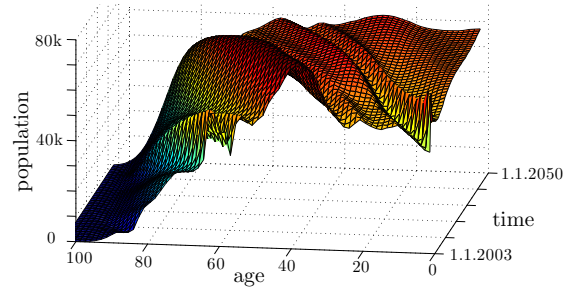
with  $\Psi(a, t) := \int_{\mathbb{R}^+} F(a, t) \beta(a, t) da$ . All  $\alpha, \beta$  and  $\gamma$  are functions of age and time and can directly be derived from [2]-[7]. The initial values are given by the initial density and the total population from [1] and [2]. Parameter  $dt$  poses for the ABM's time-step size.

The mean-field theorem guarantees asymptotic (with respect to the number of agents  $N$ ) equivalence of the PDE solution and the AB population model. Hence the ABM qualitatively behaves according to a transport-equation with additional integral-part.

Figure 1 shows a surface-plot of the resulting density for male individuals from 2003-2050. The PDE model was simulated in MATLAB using a method-of-lines scheme and its results almost perfectly **match the results of the ABM**.

## 4 Discussion

While it takes almost half a day to simulate the ABM the method of lines algorithm terminates after a few seconds. Hence it was possible to easily make quick scenario tests



**Figure 1.** Simulation results for the PDE model. Density of male population for 47 simulated years.

in order to verify the behaviour of the ABM. Also sensitivity analysis was supported: In case a parameter turned out to be very sensitive in the PDE model we paid close attention to it while analysing the ABM. Finally the 3D visualisation of the density in the PDE model inspired us to analyse diagonal cuts through the results of the ABM.

As the ABM is undoubtedly a much richer, more applicable and more flexible model, the PDE model is not intended to pose a substitute for it. Yet the time to derive, implement and execute the PDE model was still shorter than the duration of one single simulation run of the ABM in Python, which made the mean-field model a useful tool to quickly gain first insights to support validation and verification in our case.

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